Executive Summary of Minor Research Project on

Theory of Fuzzy Lie algebra over Fuzzy Field

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Lie algebra is one of the basic notions of mathematics. It is so-named in honour of Sophus Lie (1842-1899), a Norwegian mathematician who pioneered the study of this branch. The objects in Lie theory are fundamental, interesting and involving in both mathematics and physics. It has many applications to the spectroscopy of molecules, atoms and nuclei. One of the key concepts in the application of Lie algebraic method in physics is that of spectrum generating algebras and their associated dynamic symmetries. Lie algebra has also been used by electrical engineers, mainly in the mobile robot control.

According to the traditional view, science should strive for certainty(precision, specificity, sharpness, consistency etc.)in all its manifestations and hence, uncertainty regarded as unscientific. Uncertainty is an important commodity in the modeling business, which can be traded for gains in the other essential characteristics of models. This trade-off can then be utilized for constructing models that are maximally useful with respect to the purpose for which they are constructed. As a tool to study the uncertainty, in 1965, Lofti A. Zadeh developed fuzzy sets with "non-sharp boundaries".

The fuzzy set theory states that there are propositions with an infinite number of truth values, assuming two extreme values, 1 (totally true), 0 (totally false) and a continuum in between, that justify the term "fuzzy". Present day Science and Technology is featured with complex processes and phenomena for which complete information is not always available. For such cases, mathematical models are developed to handle various types of system containing uncertainty. The fuzzy sets provide us a meaningful and powerful representation of measurement of such uncertainties and vague concepts. Applications of this theory can be found, for example, in artificial intelligence, computer science, control engineering, decision theory, logic, management science, operation research and robotics.

In the beginning of 1983, K. T. Atanassov proposed one generalization of the notion of fuzzy sets, namely intuitionistic fuzzy sets [7]. He introduced the idea of defining a fuzzy set by ascribing a membership degree and a non membership degree separately in such a way that sum of the two degrees must not exceed one. This has drawn the attention of many researchers, since the intuitionistic fuzzy sets are consistent with human behaviour which reflects the hesitancy present in real life situations. Intuitionistic fuzzy set theory has been applied in different areas, such as logic programming and decision making problems.

Since its inception, the theory of fuzzy sets has advanced in a variety of ways and a lot of fuzzy mathematics has been created and developed. Theoretical advances have been made in many directions. There are fuzzy algebraic structures like fuzzy semi groups, fuzzy groups and so on. Rosenfeld [28] applied this concept to abstract algebra and formulated the notion of fuzzy subgroups and showed how some basic notions of group theory can be extended in an elementary manner to fuzzy subgroups. This object of fuzzy set theory was successively redefined and generalized by many scholars [22]. Liu [27] introduced and developed basic results concerning the notion of fuzzy subrings and fuzzy ideal of a ring. Katsaras and Liu, in their pioneering paper [20] introduced the notion of a fuzzy subspace of a vector space.

The idea of quasi-coincidence of a fuzzy point with a fuzzy set, which is mentioned in [26] has played a vital role in generating some different type of fuzzy subsystems. Using the *belong-to* relation($\in$) and *quasi-coincidence with* relation (q) between fuzzy points and fuzzy sets, the definition of (α, β) fuzzy subgroup was given by Bhakat and Das in [9]. The $(\in, \in \vee q)$ -fuzzy subgroup is an important generalization of fuzzy group. Since the algebraic structures play a prominent role in mathematics with wide range of applications, similar generalization of fuzzy subsystems of other algebraic structures is also relevant.

Fuzzy systems have been used as controllers in many applications due to their ability to use all sources of information from human experts, either numerical or linguistic. But most important issue is the stability of these systems. Lie algebra based stability analysis is one of the developing area in this regard.

The basics of fuzzy set theory explained by G. J. Klir and Bo Yuan [21], H. J. Zimmermann [32], J. J. Buckley and Esfandiar Eslami [11] helped a lot to learn more about the extension of the theory to vector spaces and Lie algebra.

The theory of Lie algebra has a group theoretic flavour. The introduction of fuzzy sets in the realm of group theory initiated the study of fuzzy subsystems of Lie algebra. In [29], Yehia extended Lie algebra to fuzzy Lie algebra over field. Akram [6] introduced (α, β) -fuzzy Lie algebras and investigated some of its properties. Fuzzy sets are intuitionistic fuzzy sets but the converse is not true. So the subsystems which are dealt with fuzzy set theory can also be dealt with intuitionistic fuzzy set theory.

The fuzzy subspaces of a vector space over fuzzy field, introduced by Malik and Mordeson [23] and the fuzzy algebra over a fuzzy field defined by Nanda [25] helped several researchers to conduct the studies of fuzzy algebraic structures over fuzzy field.

The study of fuzzy field and fuzzy algebra helped us to produce new results in the area of fuzzy Lie algebra. In [1], we introduced the concept of fuzzy Lie algebra over fuzzy field. We defined fuzzy Lie F -algebra as the fuzzy Lie algebra over a fuzzy field F . Using the concept of fuzzy Lie $F \sim$ ideal of fuzzy Lie F -algebra, fuzzy cosets of fuzzy Lie $F \sim$ ideal determined by an element of Lie algebra were also defined. We observed that there exists a Lie algebra structure on the set of all fuzzy cosets of a fuzzy Lie $F \sim$ ideal . We extended the multiplication of an element of Lie algebra by a scalar to a product of fuzzy Lie F -algebra and a scalar. Intuitionistic fuzzy Lie algebra over a fuzzy field is defined in ([3], [4]) and presented some results on it. In [2], we introduced the concept of (α, β) -fuzzy Lie algebra over an (α, β) -fuzzy field and investigated some properties. The notion of $(\in, \in \vee q)$ -fuzzy Lie $F \sim$ ideal is introduced in [5].

This dissertation on Theory of fuzzy Lie algebra over fuzzy field includes 5 chapters. Chapter 1 contains the preliminaries that are adequate for the main theory to be developed in the following chapters. The fundamentals of Lie algebra ([17],[18]) , definitions in Lattice theory [24], the results of fuzzy set theory [21], and intuitionistic fuzzy set theory [7] are included in different sections.

In Chapter 2, the notions of fuzzy subalgebras and fuzzy ideals in Lie algebra are presented and discussed important properties. Operations of fuzzy ideals are included in the last section of this chapter.

Chapter 3 presents the definition of fuzzy Lie algebra over fuzzy field. The properties of fuzzy field and fuzzy Lie algebra over fuzzy field are discussed here in detail with the help of examples. A special type of product of fuzzy Lie algebra and a scalar is also introduced. The notion of fuzzy Lie $\sim$ ideal over fuzzy field is defined and investigated the structure and properties. Fuzzy cosets are constructed to study the concept similar to quotient algebra. The last section deals with the behaviour of these structure under homomorphism.

Generalized fuzzy Lie algebras are defined in chapter 4. The definition of (α, β) -fuzzy Lie algebra over (α, β) -fuzzy field is given in the 2nd section. The relation between fuzzy Lie algebra over fuzzy field and $(\in, \in \vee q)$ -fuzzy Lie algebra over $(\in, \in \vee q)$ -fuzzy field is obtained and a characterization of $(\in, \in \vee q)$ -fuzzy Lie algebra over $(\in, \in \vee q)$ -fuzzy field is also established in this section. In the last section of this chapter, we take up the study of $(\in, \in \vee q)$ -fuzzy Lie $\sim$ ideal over fuzzy field. We obtain a characterization of $(\in, \in \vee q)$ -fuzzy Lie $\sim$ ideal and these objects are characterized by their level ideals. Here we show that every fuzzy Lie $\sim$ ideal over a fuzzy field is an $(\in, \in \vee q)$ -fuzzy Lie $\sim$ ideal over the fuzzy field.

Chapter 5 covers the theory of intuitionistic fuzzy Lie algebra over fuzzy field. The properties of this structure in terms of its membership and non-membership functions are explained in the 1st section. The behaviour of the necessity and possibility operators and the property of different level subsets are discussed. In the last section, intuitionistic fuzzy Lie $\sim$ ideal over fuzzy field is defined and several fundamental results similar to fuzzy Lie $\sim$ ideal over fuzzy field are obtained.

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