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S.I.: RELIABILITY MODELING WITH APPLICATIONS BASED ON BIG DATA

Reliability analysis of multi-state systems with common cause failures based on Bayesian network and fuzzy probability

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Abstract

Multi-state components, common cause failures (CCFs) and data uncertainty are the general problems for reliability analysis of complex engineering systems. In this paper, a method incorporating fuzzy probability and Bayesian network (BN) into multi-state systems (MSSs) with CCFs is proposed. In particular, basic theories of multi-state BN and fuzzy probability are developed. Moreover, a model integrating CCFs with BN has also been illustrated. In order to incorporate fuzzy probability into MSSs reliability evaluation considering common parent node generated by CCFs, fuzzy probability has to be translated into accurate probability through defuzzification and normalization methods which are both elaborated. In addition, quantitative analysis based on BN is carried out. In this paper, feed system of boring spindle in computer numerical control machine is analyzed as an example to validate the feasibility of the proposed method. It can improve the ability of BN on reliability evaluation of complex system with uncertainty issues.

Keywords Reliability analysis · Multi-state systems · Common cause failures · Fuzzy probability · Bayesian network

1 Introduction

The classical reliability theory, based on the assumption that events are binary, either working well or completely failed, usually ignores the effect of partial failure on system performance. However, the real system usually has multiple states varying from success to complete failure. Therefore, multi-state systems (MSSs) reliability theory has achieved great development in the recent decades.

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The research for MSSs was originated from the study conducted by Barlow & Wu, El-Neweihi, Proschan, Sethuraman, Ross, etc. (Barlow and Wu 1978; El-Neweihi et al. 1978; Ross 1979), where the basic theory and concept of multi-state were expounded. A lot of methods to analyze MSSs have been proposed by researchers. In the early stage, stochastic process methods such as Markov chain (Cafaro et al. 1986) and Semi-Markov chain (Limnios and Oprisan 2001) were applied to MSSs, followed by the methods of universal generating function (UGF) (Jafary and Fiondella 2016), binary decision diagram(Pliego Marugán et al. 2017), and fault tree analysis (Kabir 2017) which have also been put forward to reliability analysis of MSSs. Ding and Lisnianski (2008) utilized fuzzy generating function to calculate output performance distribution of MSSs. Based on binary decision diagram, Xing and Dai (2009) proposed multi-value decision diagram theory, which had great enlightening significance to future research. Nowadays with the advantage of describing multi-state events, Bayesian network (BN) has been used to MSSs research, reflecting its powerful modeling analysis capabilities (Lisnianski et al. 2017).

With the deepening of the research, common cause failures (CCFs) and relevance in MSSs have been put forward and studied by many scholars. CCFs indicate that two or more components fail due to a common cause at the same time or in a relatively short time interval. In the conventional reliability analysis, components are always been assumed to be independent. However, CCFs often play an important role in correlated failure of the parts within complex systems.

From the 1970s to now, many models and methods have been proposed for reliability analysis of binary systems subject to CCF widely, such as binomial failure rate model, basic parameter model, multiple Greek letter model (O'Connor 2013; O'Connor and Mosleh 2016). However, there may exist differences between the established reliability models and the actual situation. In the face of increasingly complex system analysis, a lot of research has been conducted to incorporate CCFs into MSSs (Ram and Manglik 2016). Many methods have been put forward to handle CCFs such as UGF method (Mi et al. 2015), optimization model (Li et al. 2010a, b), and BN (Mi et al. 2018) etc. Since BN is appropriate for uncertainty reasoning and expressing complex dependency between random variables, it has been widely used in the field of reliability analysis.

BN is a graphic modeling method which is able to calculate failure probability. Through bi-direction reasoning of BN, reliability evaluation, fault diagnosis and component importance measurement can be achieved. Li et al. (2015) investigated system reliability analysis based on dynamic fault tree and BN for systems with dynamic failure characteristics and uncertainties. Mi et al. (2016) incorporated coefficient of variation (COV) method into BN to implement reliability assessment for complex electromechanical systems under epistemic uncertainty. In addition, BN is suitable for describing MSSs and CCFs. CCFs model of system reliability based on BN has been built by Li et al. (2015). Later Mi et al. (2018) combined BN with evidence theory and formed evidential network and successfully analyzed the MSSs with CCFs.

In general, the achievements above are based on accurate probability. However, in engineering fields, accurate probability may be no longer suitable for describing the uncertainties. This may happen in general due to the factors summarized as follows: (1) unavailable of sufficient data to express the statistical properties properly; (2) inaccurate field or experiment data due to error in measurement; and (3) built model possesses fuzziness due to simplification and approximation etc. In a word, in practical engineering, it is difficult to get accurate probabilities of variables in BN because of complexity of system and incompleteness of data. To overcome these problems, a popular method known as fuzzy probability is a recently used to replace the accurate probability.

Fuzzy set theory, first proposed by Zadeh in 1965, was used to deal with the issues of imprecise or fuzzy events, and after that it has been widely used in reliability engineering (Dong et al. 2006; Lu et al. 2010; Ding et al. 2008; Cheng and Yao 2012; Ma et al. 2012) etc. Li et al. (2018b) utilized multi-source information fusion to propose a physics of failure-based reliability prediction method to deal with uncertainties. Reliability evaluation for complex engineering systems with dynamic failure mechanism and redundant architecture has also been investigated (Zhang et al. 2018; Li et al. 2018a). Since it is hard to get the accurate failure probability, so incorporating fuzzy set theory into reliability analysis enables reliability theory to handle both random and fuzzy phenomenon in practical engineering. Accordingly, many researchers have done a lot of work incorporating fuzzy set theory into BN. Dong et al. (2006) proposed set cut with confidence to describe fuzzy phenomenon in system based on BN. Lu et al. (2010) combined fuzzy set theory with BN and applied it to subway fire risk prediction. However, these investigations on reliability only analyzed binary systems. Moreover, the general definitions and properties of fuzzy MSSs were firstly proposed in Ding et al. (2008). Cheng and Yao (2012) and Ma et al. (2012) analyzed the MSSs based on fuzzy probability without the consideration of CCFs.

MSSs, CCFs and fuzzy information are all serious, important and practical factors in reliability analysis. Therefore, this paper studies the MSSs reliability analysis based on BN considering fuzzy probability and CCFs. This paper is organized as follows. Section 2 presents multi-state BN and procedures for CCF modeling. The procedures of processing fuzzy probability are introduced in Sect. 3. The proposed method is applied to feed system of boring spindle as an example in Sect. 4 and the quantitative analysis is implemented in Sect. 5. Conclusions are provided in Sect. 6.

2 Multi-state Bayesian network and CCF reliability modeling

2.1 Multi-state Bayesian network

Bayesian network, also known as belief network, is the extension of Bayes theorem which is also one of the most effective theoretical models for uncertain information representation and reasoning. Putting forward by professor Pearl from University of California in 1986, BN is a directed acyclic graph (DAG) consisting of nodes and directed acyclic arcs (Li et al. 2018b; Bobbio et al. 2001). Nodes represent variables and arcs between pairs of nodes represent the relationships between the corresponding variables. The nodes without parents are called root nodes and possess prior probabilities. All the other nodes possess conditional probability tables (CPTs) in which the nodes without descendants are called leaf nodes. A conditional probability table of a node contains the probability of each state of the node on the condition of all the combinations of the states of its parents (Lisnianski et al. 2017; Ebrahimi and Daemi 2010).

Bayesian network is based on the well-defined Bayes Rule. Assume that A and B are random variables. The condition probability of A on the condition of the state of B can be expressed as

$$P(A|B) = \frac{P(A)P(B|A)}{P(B)} \tag{1}$$

where $P(A)$ is prior probability, $P(B|A)$ is likelihood ratio and $P(A|B)$ is posterior probability.

If A is a multi-state variable owning l states a_1, a_2, …, a_l, according to Total Probability Theorem, the probability of B can be expressed as

$$P(B) = \sum_{i=1}^{l} P(B|A = a_i)P(A = a_i) \tag{2}$$

Suppose a BN possesses many variables which are expressed as $U = \{X_1, X_2, \ldots, X_n\}$, X_i ($i = 1, 2, \ldots, n$) is a failure event of a certain component or the system to be analyzed. To simplify the joint probability distribution, conditioned independency assumption is the key premise. Assume that $Pa(X_i)$ is the parent set of X_i and $Nd(X_i)$ is the non-descendant set of X_i. When conditioned on $Pa(X_i)$, X_i is independent with $Nd(X_i)$.

Therefore, we can get Eq. (3).

$$P(X_i|Pa(X_i), Nd(X_i)) = P(X_i|Pa(X_i)) \tag{3}$$

According to conditioned independency assumption, the joint probability distribution can be obtained as

$$P(U) = P(X_1, X_2, \ldots, X_n) = \prod_{i=1}^{n} P(X_i|Pa(X_i)) \tag{4}$$

Then, the probability distribution of X_i can be calculated as

$$P(X_i) = \sum_{\text{except}(X_i)} P(U) \tag{5}$$

Given the evidence E (i.e., the states of another set of nodes are known with certainty), the posterior probability distribution of nodes can be computed as

$$P(U|E) = \frac{P(U, E)}{P(E)} = \frac{P(U, E)}{\sum_{U} P(U, E)} \tag{6}$$

Since Bayesian network owns the function of bi-directional reasoning (Ozbay and Noyan 2006), it is easy to get the system failure probability from the prior probabilities and condition probability tables, and the backward reasoning also makes it convenient to evaluate the unit importance and implement fault diagnosis. In addition, it has the advantage of describing multi-state features and uncertainty of logic failure relationship.

2.2 CCF modeling based on Bayesian network

In most complex systems, some components fail because of common causes, such as design weakness, extreme environmental condition, etc. Therefore, there is relatively large error in traditional reliability analysis, which is based on the assumption that components are independent, that is, CCF is ignored.

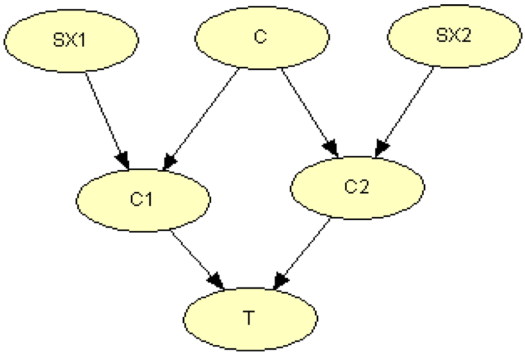
The process of building the CCF model based on BN is summarized as the following three steps (Li et al. 2015).

1. Divide the common cause components X1 and X2 into independent failure sub-components SX1, SX2 and CCF sub-component C, and the relationships among these sub-components are series;
2. Divide the failure rate of common cause components P_{X1} and P_{X2} into independent failure rate P_{SX1}, P_{SX2} and CCF rate P_C;

3. Analyze logic relationships among C1, C2 and the others and build their CPTs.

Series systems and parallel systems are the most common and basic models applied in reliability analysis. In series systems, any component's failure will lead to the system failure. And in parallel systems, the system will fail only when all the components fail.

The CCF model of series systems based on BN is shown in Fig. 1 where SX1 and C, SX2 and C are both in series. So do C1 and C2.

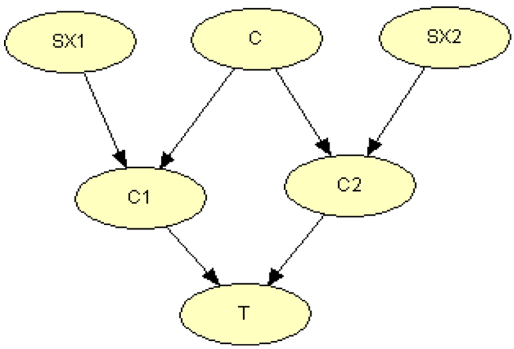


$P(C1=1\,|\,SX1=0,\ C=0)=0$ $\qquad$ $P(C2=1\,|\,SX2=0, C=0)=0$

$P(C1=1\,|\,else)=1$ $\qquad$ $P(C2=1\,|\,else)=1$

$$P(T=1\,|\,C1=0,\ C2=0)=0$$
$$P(T=1\,|\,\text{else})=1$$

Fig. 1 CCF model of series system based on BN



$P(C1=1\,|\,SX1=0,\ C=0)=0$ $\qquad$ $P(C2=1\,|\,SX2=0, C=0)=0$

$P(C1=1\,|\,else)=1$ $\qquad$ $P(C2=1\,|\,else)=1$

$$P(T=1\,|\,C1=1,\ C2=1)=1$$
$$P(T=1\,|\,\text{else})=0$$

Fig. 2 CCF model of parallel system based on BN

The CCF model of parallel systems based on BN is shown in Fig. 2 where C1 and C2 are in parallel while SX1 and C, and SX2 and C are series systems. Here, T denotes 'failure of the series system'.

State 1 represents the component failure and state 0 represents the component working in both figures.

3 The processing of fuzzy probability

For some MSSs, besides the above mentioned three conditions, the boundary determination between different states is also fuzzy due to the multiple and dependent failure modes of mechanical components, such as corrosion, abrasion, etc. In addition, a complex system may fail due to many different causes at the same time, for which it is difficult to determine relationship between a failure and corresponding causes. Moreover, human factors are also important reason of incomplete data. Therefore, such fuzziness in a system has to be addressed to obtain the accurate failure probabilities in practical engineering by applying fuzzy probabilities.

3.1 Fuzzy probability

In reliability analysis, accurate probability of each state is usually difficult to obtain. To address this problem, fuzzy number is used to represent the occurrence probability of each event in this paper. Fuzzy number is not a single value but a set of possible values in which every value owns a weight number in between 0 and 1. Each weight, representing the membership degree of each value, is decided by membership function.

There are many forms of fuzzy number, where triangular fuzzy number and normal fuzzy number are two common choices. Because of the advantage of convenient operation, the triangular fuzzy number is adopted in this paper. The membership function of triangular fuzzy number shown in Fig. 3 can be expressed as (Lee et al. 2012; Lan and Fan 2010)

$$\mu(x) = \begin{cases} 0, & x < a \\ (x-a)/(m-a), & a \le x < m \\ (b-x)/(b-m), & m \le x \le b \\ 0, & x > b \end{cases} \tag{7}$$

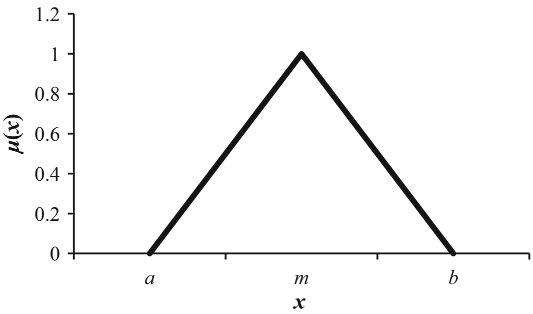


Fig. 3 Triangular membership function

Hence the triangular fuzzy number can be represented by three parameters a, m and b, and denoted as (a, m, b).

For triangular fuzzy numbers $A = (a_1, m_1, b_1)$ and $B = (a_2, m_2, b_2)$, the operation algorithms are as follows.

The sum of two fuzzy numbers is expressed as

$$A \oplus B = (a_1 + a_2, m_1 + m_2, b_1 + b_2) \tag{8}$$

The product of two fuzzy numbers is expressed as

$$A \otimes B = (a_1 \times a_2, m_1 \times m_2, b_1 \times b_2) \tag{9}$$

The division of two fuzzy numbers A and B for B not equal to zero can be expressed as Eq. (10).

$$A/B = \left(\frac{a_1}{a_2}, \frac{m_1}{m_2}, \frac{b_1}{b_2}\right) \tag{10}$$

3.2 Processing of fuzzy probability

While incorporating fuzzy probability into BN, the situation with common parent node is always avoided. However, according to Section 2, the nodes C1 and C2 in BN with CCF have a common parent node C. Therefore, the fuzzy probability needs to be processed before quantitative analysis. In particular, the fuzzy failure probabilities of basic failure events, or root nodes in BN model, are obtained by analyzing the incomplete data in field, and then the defuzzification and normalization can be carried out according to the following method before BN reasoning.

3.2.1 Defuzzification

In the fuzzy set, we can select an accurate number which can best represent the fuzzy set relatively. Such process is called defuzzification. The methods of defuzzification come in many forms such as mean area method, weighted average method, maximum membership degree, gravity method, etc.

Here using the mean area method (Ma et al. 2012), the fuzzy probability is replaced by an accurate probability. The probability of variable X_i in state j is

$$P'_{ij} = \frac{a_{ij} + 2m_{ij} + b_{ij}}{4} \tag{11}$$

3.2.2 Probability normalization

In order to satisfy the normalization condition that the sum of probabilities of one variable at different states equals to one, the accurate probabilities of states of root nodes obtained by defuzzification need to be normalized. Assume variable X_i has r_i states, the accurate probability of variable X_i in state j is

$$P''_{ij} = \frac{P'_{ij}}{\sum_{j=1}^{r_i} P'_{ij}} \tag{12}$$

4 Reliability analysis of boring spindle feed system

As an important index of numerical control machine, reliability has been a serious problem perplexing computer numerical control (CNC) machine industry and has become the focus of market competition (Zhang 2012). Boring spindle is an important component in CNC machine whose feed system determines its working quality.

4.1 Feed system of boring spindle

As a key part of implementing the movement of boring spindle, its feed system is prone to fail. Therefore, it is greatly significant to analyze its feed system. The mechanical parts of the feed system are divided into 3 subsystems: servo motor (SM), gear drive (GD) and ball screw pair drive (BSPD). The feed system is driven by servo motor. The movement of boring spindle is realized by driving ball screw pair after gear reducer. The gear drive (GD) consists of two gears (G) and a ring flange (RF). The ball screw pair drive (BSPD) consists of ball screw pair (BSP), bearings (B) and lock nuts (LN).

The reliability block diagram of feed system is shown in Fig. 4. Any component's failure will cause the system failure.

The operation of machine and work environment will bring much quantity of heat which could cause deformation and agglutination of components and thus has great impact both on gear drive and ball screw pair drive.

Using the method proposed in Sect. 2, the BN of the feed system considering CCF is mapped as shown in Fig. 5.

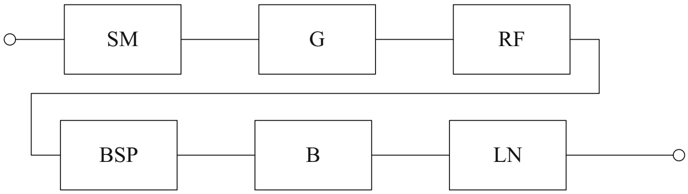


Fig. 4 Reliability block diagram of feed system

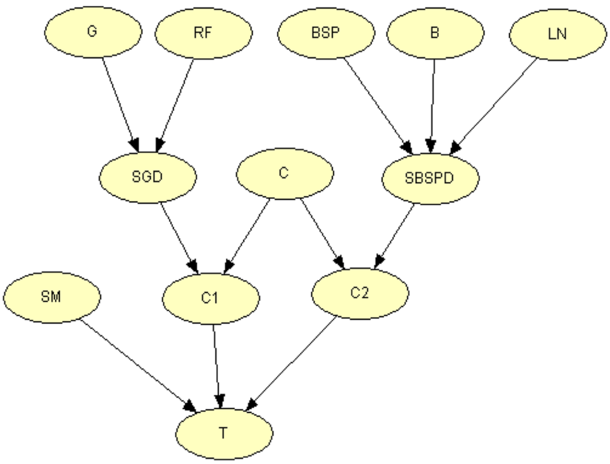


Fig. 5 Bayesian network of feed system

The node SGD represents the independent failure subsystem of gear drive and the node SBSPD represents the independent failure subsystem of ball screw pair drive. The node C represents the common cause, i.e., heat.

Suffering from friction, loading force, etc., gears, ball screw pair and bearings inevitably experience abrasion, deformation and fatigue which would decrease the positioning accuracy of these components and make them fail completely. Therefore, this paper considers gears, ball screw pair and bearings with 3 states respectively: working well, partial failure and failure. Servo motor, ring flange and lock nuts have 2 states: working and failure. Heat would also decrease the positioning accuracy of these two subsystems and when the heat reaches a certain degree, the subsystems will fail completely. So this paper considers the heat with 3 grades: 1, 2 and 3. Besides, Gear drive and ball screw drive both have 3 states. So does the whole system. State 0, 1, and 2 represent working well, partial failure, and failure, respectively. The conditional probabilities are shown in Tables 1, 2, 3, 4 and 5.

Table 1 Conditional probability table of node SGD

G	RF	SGD = 0	SGD = 1	SGD = 2
0	0	1	0	0
0	2	0	0	1
1	0	0	1	0
1	2	0	0	1
2	0	0	0	1
2	2	0	0	1

Table 2 Conditional probability table of node SBSPD

BSP	B	LN	SBSPD = 0	SBSPD = 1	SBSPD = 2
0	0	0	1	0	0
0	0	2	0	0	1
0	1	0	0	1	0
0	1	2	0	0	1
⋮	⋮	⋮	⋮	⋮	⋮
2	2	0	0	0	1
2	2	2	0	0	1

Table 3 Conditional probability table of node C1

SGD	C	C1 = 0	C1 = 1	C1 = 2
0	0	1	0	0
0	1	0	1	0
0	2	0	0	1
⋮	⋮	⋮	⋮	⋮
2	0	0	0	1
2	1	0	0	1
2	2	0	0	1

Table 4 Conditional probability table of node C2

SBSPD	C	C2 = 0	C2 = 1	C2 = 2
0	0	1	0	0
0	1	0	1	0
⋮	⋮	⋮	⋮	⋮
2	0	0	0	1
2	1	0	0	1
2	2	0	0	1

Table 5 Conditional probability table of node T

SM	C1	C2	T = 0	T = 1	T = 2
0	0	0	1	0	0
0	0	1	0	1	0
0	0	2	0	0	1
0	1	0	0	1	0
0	1	1	0	1	0
0	1	2	0	0	1
⋮	⋮	⋮	⋮	⋮	⋮
2	2	0	0	0	1
2	2	1	0	0	1
2	2	2	0	0	1

Table 6 Fuzzy probability of root node

Code	0	1	2
SM	(0.8991, 0.9594, 0.9914)	–	(0.0138, 0.0151, 0.0162)
G	(0.8981, 0.9672, 0.9912)	(0.0174, 0.0204, 0.0224)	(0.0108, 0.0124, 0.0137)
RF	(0.9035, 0.9946, 0.9981)	–	(0.0047, 0.0054, 0.0059)
BSP	(0.8812, 0.9571, 0.9889)	(0.0255, 0.0295, 0.0325)	(0.0119, 0.0134, 0.0147)
B	(0.8775, 0.9475, 0.9871)	(0.0266, 0.0316, 0.0346)	(0.0178, 0.0209, 0.0230)
LN	(0.9058, 0.9958, 0.9981)	–	(0.0037, 0.0042, 0.0046)
C	(0.8919, 0.9609, 0.9819)	(0.0210, 0.0251, 0.0276)	(0.0120, 0.0140, 0.0154)

Considering the three conditions resulting in fuzziness and the fuzzy multiple state boundaries, the fuzzy probability of each root node in each state shown in Table 6 can be obtained from statistics and analysis of failure data of feed system of boring spindle.

4.2 The processing of fuzzy probability

4.2.1 Defuzzification

The method of defuzzification has been introduced as above. By defuzzification, accurate probabilities can be obtained from fuzzy probability so that it is easy to handle the repetitive

Table 7 Defuzzification and normalization of root node

Code	Defuzzification			Normalization		
	0	1	2	0	1	2
SM	0.9523	–	0.0150	0.9845	–	0.0155
G	0.9567	0.0202	0.0123	0.9672	0.0204	0.0124
RF	0.9727	–	0.0053	0.9946	–	0.0054
BSP	0.9461	0.0292	0.0133	0.9570	0.0295	0.0135
B	0.9399	0.0311	0.0207	0.9478	0.0313	0.0209
LN	0.9739	–	0.0042	0.9957	–	0.0043
C	0.9489	0.0247	0.0139	0.9609	0.0250	0.0141

bottom event caused by CCF. According to Eq. (8), defuzzification can be achieved and the results are shown in Table 7.

4.2.2 Probability normalization

The accurate probabilities obtained from fuzzy probabilities need to be normalized and the normalization method has been introduced above. According to Eq. (9), the eventual probabilities can be obtained, as shown in Table 7.

5 Quantitative analysis based on bayesian network

5.1 Bi-direction reasoning of BN

According to the theory of multi-state BN, the marginal probability distribution of leaf node T corresponding to the occurrence probability of system can be obtained easily. Here, the states 0, 1 and 2 are corresponding to working state, partially failure state and failure state of the system respectively, and the probability values are as follows.

$$P(T = 0) = 0.8219$$
$$P(T = 1) = 0.0950$$
$$P(T = 2) = 0.0831$$

In addition, the posterior probabilities of root nodes in each state conditioned on the value of leaf node T are also easily achieved. The results are shown in Table 8. The posterior probabilities of root nodes in each state while the leaf node is in state 1 and 2 are graphically listed in Figs. 6 and 7, respectively.

5.2 Importance analysis

Importance, which indicates important degree of each component in system aiming at quantifying the contributions the component failure making to the system failure, is an important part of quantitative analysis in system reliability analysis (Zeng and Guo 1999; Kuang and Xie 2013). In MSSs, probability importance reflects the influence of all failure states of node X_i on the state J of leaf node T.

Table 8 Posterior probabilities of root nodes

Code		T = 1	T = 2	Code		T = 1	T = 2
SM	0	1	0.8134	BSP	0	0.7114	0.8125
	2	0	0.1866		1	0.2886	0.0250
G	0	0.8007	0.8332		2	0	0.1625
	1	0.1993	0.0175	B	0	0.6915	0.7245
	2	0	0.1493		1	0.3085	0.0239
RF	0	1	0.9350		2	0	0.2516
	2	0	0.0650	C	1	0.7553	0.8092
LN	0	1	0.9483		2	0.2447	0.0211
	2	0	0.0517		3	0	0.1697

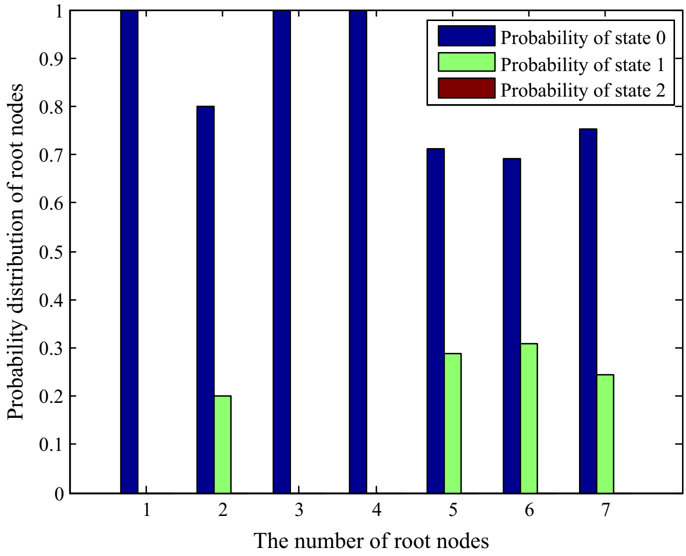


Fig. 6 The probability distribution for each root nodes in state 1

Assume that root node X_i has r_i states. When the state of leaf node is known with certainty, probability importance of root node X_i is

$$I_{T_J}^{\mathrm{Pr}}(X_i) = \sum_{j=0}^{r_i-1} [P(T = J|X_i = j) - P(T = J|X_i = 0)] \tag{13}$$

According to Eq. (10), the probability importance of each root node can be obtained and is shown in Table 9.

According to Table 9, nodes SM, RF and LN have no influence on the occurrence of state 1 of leaf node T. Node B has the biggest influence on the occurrence of both state 1 and state 2 of leaf node T. Nodes BSP and C follows it. According to the order of the posterior probabilities of each root node, the system fault diagnosis and maintenance detection can be performed efficiently. The components having the most importance degrees should be detected and maintained first.

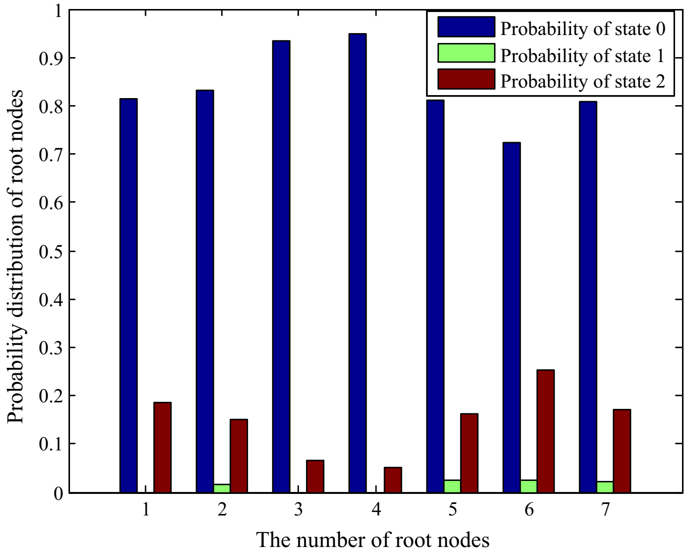


Fig. 7 The probability distribution for each root nodes in state 2

Table 9 Probability importance of root nodes

Code	$T = 1$	$T = 2$
SM	–	0.9314
G	0.7710	0.9284
RF	–	0.9219
BSP	0.7883	0.9295
B	0.7979	0.9365
LN	–	0.9209
C	0.7806	0.9300

6 Conclusions

Bayesian network is a popular probabilistic method that can be used to describe complex dependency and implement uncertainty reasoning between random variables. Also, as discussed multi-state components, CCFs and fuzzy probability are very important to be address in reliability analysis. Accordingly, this paper builds the multi-state BN with CCFs for reliability analysis and incorporates fuzzy probability into the modeling to handle system's fuzziness in practical engineering. In particular, the defuzzification and normalization methods are applied to process the fuzzy probability. Consequently, it gets the accurate probability from fuzzy probability, and analyze the multi-state BN with CCFs. This method can easily handle the common parent node caused by CCFs in general fuzzy operators.

The feed system of boring spindle in CNC machine is considered which has both multi-state components and CCFs because of its failure mechanism, work environment, etc. By analyzing the failure causes of the system, its reliability block diagram has been established. Considering the multi-state components and CCF, multi-state BN of feed system has also been built. Then, the defuzzification and normalization of the fuzzy probabilities of root nodes are carried out to get the accurate probabilities. According to the conditional probability tables and accurate probabilities of root nodes, the quantitative analysis containing bi-directional reasoning and importance analysis are implemented. Thus, the reliability analysis of feed system is achieved and the feasibility of the proposed method is validated. It improves the ability of BN to solve uncertainty issue and complex system and extends application field of BN.

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