St. Thomas College (Autonomous), Thrissur

Proceedings of

NATIONAL VIRTUAL CONFERENCE ON EMERGING TRENDS IN APPLIED MATHEMATICS

December 16-18, 2020

Publications Division, St. Thomas College Thrissur

Dr. Sr. Julie Andrews (Chief Editor)
Sabu A S
Renjith Varghese

PG & Research
Department of Mathematics
with
IQAC, St. Thomas College (Autonomous), Thrissur

PROCEEDINGS OF NATIONAL VIRTUAL CONFERENCE ON

EMERGING TRENDS IN APPLIED MATHEMATICS

16-18, DECEMBER 2020

ETAM-2020

Abstracts

Organized by

PG & RESEARCH DEPARTMENT OF MATHEMATICS

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ST. THOMAS COLLEGE (AUTONOMOUS), THRISSUR

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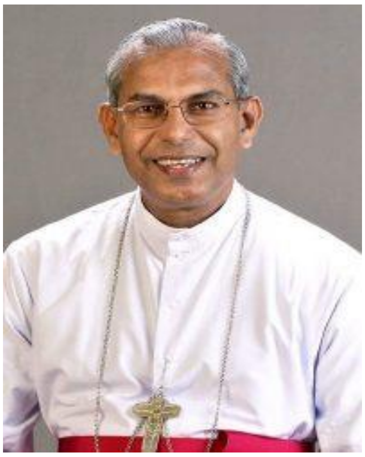

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Although every effort has been taken to avoid errors while publishing the Abstracts, in advertent errors may please be omitted

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Editorial

Veritas Vos libera bit

It gives immense joy to present before you the Proceedings of National Virtual Conference on Emerging Trends in Applied Mathematics, December 16-18, 2020, even though in the midst of Covid-19 pandemic, organized by the Research and Post Graduate Department of Mathematics, St. Thomas' College (Autonomous), Thrissur. We are so happy to bring out this issue which features paper presentations of the conference. I take this opportunity to wholeheartedly thank God, the Almighty and St. Thomas, the apostle of India and all my colleagues and friends, and each one, who made this humble attempt a success.

With warm personal regards and high esteem

Dr. Sr. Julie Andrews (Chief Editor)

ABOUT THE COLLEGE

St. Thomas' College was founded as an educational institution in 1889 by the then Vicar Apostolic of Thrissur, Rt. Rev. Adolphus Edwin Medlycott, PhD, a former Professor of Rhetoric at the Propaganda College, Rome. The College began as a High School in 1894 under the managership of Fr. Zambanelli, and Fr. Paul Alappatt, Ph.D. was appointed as the Manager and Rector of the institution in 1895. The University of Madras gave formal affiliation in Group III in April 1919 and the St. Thomas' High School formally became a Second Grade College with Fr. John Palocaren as the first Principal. The Junior Intermediate College started on 8th June 1919 with 96 students and 5 teachers. The College rose to the status of a First Grade College in History Group in March 1925. With the formation of Universities in Kerala, the College was affiliated to the University of Kerala in 1957 and to the University of Calicut in 1968. St. Thomas' College was accredited for the second time in 2010 by the National Assessment and Accreditation Council with A grade for a period of five years with CGPA of 3.58 on a four point scale. In the third Cycle of NAAC accreditation, the college was granted A Grade with 3. The UGC granted Autonomous Status to the College on 13.06.2014. The UGC confirmed St. Thomas College with the status of 'College with Potential for Excellence'. In the NIRF Ranking 2018, the college was positioned nationally at Rank 79. In the NIRF Ranking 2019, the college was ranked at 54. The College continuously engages the suggestions, demands and requests through a wide variety of institutional mechanisms such as Academic Council, Boards of Studies, Parent-Teacher Association, IQAC, Department Councils, College Council, College Union, etc. The College has 23 departments, 14 PG programmes 23 UG programmes and 9 research centres. Alumni association, PTA and Management of the College are active in promoting the academic and administrative aspects of the College. In 2019, St. Thomas College was chosen as a Mentor College to help the unaccredited colleges to get accredited through the 'Paramarsh Scheme'.

ABOUT THE DEPARTMENT

B.A. Course in Mathematics under Madras University was started in June 1926. It was then converted to B.Sc. Degree course in Mathematics under the University of Kerala in June 1957. M.Sc. course was started in June 1964. The Special B.Sc. Degree Course in Mathematics was started in June 1966 and was discontinued in 1970. The combined Department of Mathamatics and Statistics was bifurcated in 1984, with the introduction of M.Sc. in Statistics. Late Prof. V. Subbaraman was the first Head of the Department of Mathematics. Rev. Fr. (Dr.) Thomas Moothedan D.D., the noted Biblical scholar and translator of the first authorised version of the Catholic edition of the Bible in Malayalam, was a member of the Department and later became the Principal of the College from 1961 to 1971. Noted mathematician Shri. V. K. Krishnan Ph.D, whose Functional Analysis and Fundamentals of Real Analysis have been published respectively by the Prentice-Hall and Pearson Education, was a member of the Department. Eminent teachers like K. M. Antony, A.T. Antony, K.V. Ignatious, V.R. Krishna Chandran (an authority on the traditional Kerala theatre and whose Purusarthakuthu is published by the Kerala Sahitya Academy),C. Balakrishnan, K. Achutha Warrier, S.Jayaram, Alosius Boustine, K. M. Thomas, Thomas K. Kuruvila, E.K. Jose, C.D. Paul, M.D. Paul, T.P. Jose, M.D. Varghese, P. L. Antony , C. I. Thomas and Vincent Joseph Pulikkottil served the Department. Dr.Thomas Moothedan memorial endowed lecture is conducted on applied mathematics from 2013 onwards. Mathematics Students Journal Math-Wissen is being published by M.Sc. Students from 2013. Department organizes state level Math-Puzzle Quiz competitions every year in memory of Prof. M. D. Paul, who died in service on 27-4- 2012. Dr. P .L. Antony was the first controller of examinations and was a vice principal of the college. The Department was recognized as a Research Centre by the University of Calicut on 26th march 2015.The department offers Certificate course in Programming with PYTHON for UG and Certificate course in MATLAB for PG students.

NATIONAL VIRTUAL CONFERENCE ON EMERGING TRENDS IN APPLIED MATHEMATICS

16-18, DECEMBER 2020

ETAM-2020

INVITED TALKS – ETAM 2020

	Dr. S P Anjali Devi Rtd. Professor and Head Department of Applied Mathematics Bharathiar University, Coimbatore, TN Inauguration: Talk on Emerging Trends in Applied Mathematics
	Dr. Paul Isaac Associate Professor and Research Guide Dept. Of Mathematics Bharata Mata College, Kochi Invited Talk: An Introduction to Fuzzy Set Theory
	Dr. Satyananda Panda Dept. of Mathematics National Institute of Technology Calicut Invited Talk : Introduction of a model to describe the bacteria decontamination by the disinfectant on a hard surface
	Dr. Fr. Joseph Kuritheera CMI Dean of Research, Associate Professor in Mathematics Christ University, Bengaluru, Karnataka Invited Talk: Graph Theory and Epidemiology

NATIONAL VIRTUAL CONFERENCE ON EMERGING TRENDS IN APPLIED MATHEMATICS-2020 (ETAM-2020)

Programme Shedule for ETAM-2020

16 December 2020

1.30 to 2 p.m.

Inauguration

2.00 to 4.00 p.m.

Talk 1	:	Dr S P Anjali Devi (Former Professor & Head, Department of Applied Mathematics,Bharathiar University,Coimbatore)

4.00 to 6.00 p.m.

Talk 2	:	Rev. Dr. Joseph Varghese Kureethara (DoR, Christ University, Bangalore) Topic: Graph Theory and Epidemiology

6.00 to 7.00 p.m

Paper Presentation

NATIONAL VIRTUAL CONFERENCE ON EMERGING TRENDS IN APPLIED MATHEMATICS-2020 (ETAM-2020)

Programme Schedule for ETAM-2020

17 December 2020

2.00 to 4.00 p.m.

Talk 3 : Dr. Paul Isaac (Head, Department of Mathematics, Bharatamata College, Thrikkakkara) Topic: An Introduction to Fuzzy Set Theory

4.00 to 6.00 p.m.

Paper Presentation

18 December 2020

2.00 to 4.00 p.m.

Talk 3 : Dr. Sathyananda Panda (Head, Department of Mathematics, NIT, Calicut) Topic: Surface Decontamination: A Mathematical Model

4.00 to 5.00 p.m.

Valedictory function

CONTENTS

INVITED TALK -1

GRAPH THEORY AND EPIDEMIOLOGY

DR. JOSEPH VARGHESE KUREETHARA,

DOR, Christ University, Bangalore

An epidemic is a disease that affects a sizable number of people due to their interactions with each other. Graph is a representation of people (objects) and their relationships. In graph theory an object is a vertex and two objects that are in immediate contact to each other are represented by a line (edge) between them. In the scientific study of epidemics, epidemiology, several Mathematical models are used. Graph models such as Zachary's Karate Club, Community, Small-world, Six-degrees of Separation, hypergraphs etc. are successfully employed in epidemiological studies around the world.

INVITED TALK -2

AN INTRODUCTION TO FUZZY SET THEORY

DR. PAUL ISAAC

Head, Department of Mathematics, Bharatamata College, Thrikkakkara

The talk introduces Fuzzy set theory, with certain important definitions and theories. The intuitionistic fuzzy sets are also described in brief. Also, the applications of the topic with eminent scientists' contribution are dealt with.

INVITED TALK -3

SURFACE DECONTAMINATION: A MATHEMATICAL MODEL

DR. SATHYANANDA PANDA

Department of Mathematics, NIT, Calicut

Model to describe the bacteria decontamination by the disinfectant on a hard surface. The model is a reaction-diffusion type with boundary and initial conditions. The model equation is solved using the finite volume numerical method. The results reveal that the bacteria may grow or decay depending upon the process parameters.

ADVANCED ENCRYPTION STANDARD-A TOOL FOR SYMMETRIC ENCRYPTION

ABOOBACKER P

Assistant Professor, Department of Mathematics, Govt. Engineering College, Palakkad

Email: backer83@gmail.com

ABSTRACT

With the fast evolution of digital data exchange, security of information becomes much important in data storage and transmission. A desirable property of any cryptographic algorithm is the resistance against various attacks. One of the methods to strengthen the cipher against the attacks is the use of Substitution-Permutation Networks (SPN) to exploit the non-linearity of Substitution boxes(S-Box). The Advanced encryption Standard (AES) is a substitution-permutation Network which operates over the Galois field GF(2^8).The S-box make use of the algebraic properties of the finite field which in turn has its own drawbacks. This paper is a review of the substitution-permutation networks especially the Advanced Encryption Standard (AES) emphasizing on the Mathematical aspects behind the design criterion.

Keywords- S-box, AES, Block cipher,SPN

CATALAN NUMBERS AND THEIR APPLICATIONS

JOHNY THOMSON

B.Sc. Scholar, Department of Statistics, St. Thomas' College (Autonomous), Thrissur, Kerala, India. Email:johnyttofficial@gmail.com

ABSTRACT

In this paper, we try to define the Catalan numbers with the help of the famous Parenthesis problem. Firstly, we try to familiarize ourselves with the tools and formulas we'll be using to derive the formula for the n^{th} Catalan number, C_n. Then We'll try to use a simple transformation technique which will make the counting not only possible but also easier and finally come up with a closed formula. We'll also look at some of their applications in Mathematics and in real life, specifically in Cryptography and Computer Programming.

Key words: Catalan Numbers, Permutation with Repetition, Bijection Principle, Parathesis, Dyck Paths

1. INTRODUCTION – A BRIEF HISTORY

The Catalan numbers are one of the most important sequences of combinatorial numbers, with a large range of occurrences in apparently different counting problems. In the modern mathematical literature, Catalan numbers are wonderfully ubiquitous. Although they appear in a variety of disguises, we are so used to having them around, it is perhaps hard to imagine a time when they were either unknown or known but obscure and underappreciated.

In 1751, Leonhard Euler (1707–1783) introduced and found a closed formula for what we now call the Catalan numbers. The proof of this result had eluded him, until he was assisted by Christian Goldbach (1690–1764), and more substantially by Johann Segner. By 1759, a complete proof was obtained. Euler defines Catalan numbers C_n as the number of triangulations of (n + 2)-gon, and gives the values of C_n for $n \leq 8$ (evidently, computed by hand). All the values he deduced then, including $C_8 = 1430$ are correct.

In 1988, it came to light that the Catalan number sequence had been used in China by the Mongolian mathematician Mingantu by 1730. For instance, Ming used the Catalan sequence to express series expansions of sin(2α) and sin(4α) in terms of sin(α) in his book *Ge Yuan Mi Lu Jie Fa [The Quick Method for Obtaining the Precise Ratio of Division of a Circle]*

In 1838, French4 and Belgium mathematician Eug`ene Charles Catalan (1814–1894) was a r´ep´etiteur at Ecole Polytechnique, and a former student of Liouville. He was the first to obtain what are now standard formula;

$C_n = 2n! / n! (n + 1)!$

He then studied the problem of computing the number of different (non-associative) products of n variables, equivalent to counting the number of bracket sequence. Later on, mathematicians were able to come up with an interesting recurrence relation for these numbers;

$C_n+1=C_0C_n+C_1C_{n-1}+\cdots+C_nC_0$

2. PERMUTATION WITH REPETITION

If the objects are all distinct, then we know that the number of permutations without repetition is n! For each of these permutations, we can permute the n_1 identical objects of type 1 in n_1! possible ways; since these objects are considered identical, the arrangement is unchanged. Similarly, we can take any of the n_2! permutations of the n_2 identical objects of type 2 and obtain the same arrangement. Continuing this argument, we account for these repeated arrangements by dividing by the number of repetitions. Hence, the number of permutations of n objects with n_1 identical objects of type 1, n_2 identical objects of type 2…, and n_k identical objects of type k is:

$n!/n_1!n_2!...n_k!$

3. BIJECTION THEOREM

If a function f:A→B is one-one and onto, the function is termed a bijection hence, the cardinalities of both sets are equal, i.e. ,

$|A| = |B|$

This is very useful when we can't count something directly but can map every element bijectively to another set which's easier to count.

4. THE PARENTHESIS PROBLEM

In this famous problem, we're challenged to find the total number of valid parenthesis expression made with only n opening brackets ([ ) and n closing brackets ( ]). In a valid expression, the opening brackets are equally balanced out by closing brackets and no parenthesis goes un-closed. The answer and its proof and derivation has got many versions including those which use recursions and the bijection principle. Here, we'll be attacking the problem with the help of the bijection theorem.

4.1 Method of Derivation:

- Represent an open bracket by +1 and a closed bracket by -1.
- Expression becomes invalid at the stage where we close an un-opened bracket; i.e., when number of -1s ( ]) becomes more than the +1s ([ ).

Now we aim to find the number of invalid expressions.

- Consider set of invalid expressions A = $a_1\, a_2\, a_3 \ldots\ldots a_{k-1}\, a_k\, a_{k+1} \ldots\ldots a_{2n},$ where $a_i = \pm 1$.
- Let k be the smallest state where an extra unbalanced -1 ( ]) is introduced.
- Now note the following:
  - $a_k = -1$
  - $a_1\, a_2\, a_3 \ldots\ldots a_{k-1}$ has equal number of +1s and -1s, say j
  - $a_{k+1} \ldots\ldots a_{2n}$ has *(n-j)* +1s and *(n-(j+1))* -1s
- Now we'll create a new family of sequences B= $a_1\, a_2\, a_3 \ldots\ldots a_{k-1}\, a_k\, b_{k+1} \ldots\ldots b_{2n}$; where $b_i = -a_i$, $k+1 \leq i \leq 2n$.
- Note that since we reversed the signs of every ai after ak in B, $a_1\, a_2\, a_3 \ldots a_{k-1}\, a_k\, b_{k+1} \ldots b_{2n}$ now has *(n-j)* -1s and *(n-(j+1))* +1s.
- Now we have *(n-1)* +1s and *(n+1)* -1s in total in a sequence of B. Since the number of closed brackets more than open brackets, every expression in B is going to be an invalid one.

4.1.1 Showing this Mapping is Bijective:

- If 2 different sequences Ai and Aj , have same transformed image in B ;

$\{ a_1 a_2 a_3 \ldots\ldots a_{k-1} a_k b_{k+1} \ldots.. b_{2n} \}=\{ a'_1 a'_2 a'_3 \ldots\ldots a'_{k-1} a'_k b'_{k+1} \ldots.. b'_{2n} \}$

After reverse transformation by changing every bi and b'i back to their respective ai,

$\{a_1 a_2 a_3 \ldots\ldots a_{k-1} a_k a_{k+1} \ldots. a_{2n}\} = \{a'_1 a'_2 a'_3 \ldots\ldots a'_{k-1} a'_k a'_{k+1} \ldots. a'_{2n}\}$

Hence Ai = Aj; which implies that the mapping is one-one.

- For every sequence in B, reverse transformation yields a sequence in A;

Hence the mapping is onto as well.

- Therefore, we infer that this mapping is Bijective.

4.1.2 Counting Cardinality of B:

- Number of sequences in B is simply the number of distinct permutations of *(n-1)* +1s and *(n+1)* -1s, which is:

  |B| = (2n)!/(n-1)!(n+1)!

- Since the mapping is bijective the number of invalid expressions |A| = |B|

4.2 Final Trick

Subtract |A| from total number of distinct permutations of n +1s and n -1s, to get number of valid expressions Cn, which's termed the n^{th} Catalan Number; note that total possible number of distinct expressions using n +1s and -1s is (2n)!/n!n! . Hence;

Cn = (2n)! /n!n! - (2n)! /(n-1)! (n+1)! =(2n)! /n! (n+1

5. APPLICATIONS

Catalan numbers have a significant place and major importance in combinatorics and computer science. They appear in the triangulation problem of polygon and polyhedron, binary trees, multiplication ordering, lattice path problem, etc. In fact, the n^{th} Catalan number gives us:

- Number of full binary trees (A rooted binary tree is full if every vertex has either two children or no children) with n+1 leaves.

- The number of paths with 2n steps on a rectangular grid from bottom left, i.e., (n-1, 0) to top right (0, n-1) that do not cross above the main diagonal.

- Number of ways a convex polygon of n+2 sides can split into triangles by connecting vertices.
- Number of non-crossing partitions of the set {1, …, 2n} in which every block is of size 2. A partition is non-crossing if and only if in its planar diagram, the blocks are disjoint (i.e. don't cross).
- Number of Dyck words of length 2n. A Dyck word is a string consisting of n X's and n Y's such that no initial segment of the string has more Y's than X's. For example, the following are the Dyck words of length 6: XXXYYY, XYXXYY, XYXYXY, XXYYXY, XXYXYY.
- Number of ways to connect the points on a circle using disjoint chords.
- Number of ways to form a "mountain ranges" with n upstrokes and n down-strokes that all stay above the original line. The mountain range interpretation is that the mountains will never go below the horizon.
- Number of stack-sortable permutations of {1, …, n}. A permutation w is called stack-sortable if S(w) = (1, …, n), where S(w) is defined recursively as follows: write w = unv where n is the largest element in w and u and v are shorter sequences, and set S(w) = S(u)S(v)n, with S being the identity for one-element sequences.
- Number of permutations of {1, …, n} that avoid the pattern 123 (or any of the other patterns of length 3); that is, the number of permutations with no three-term increasing subsequence. For n = 3, these permutations are 132, 213, 231, 312 and 321. For n = 4, they are 1432, 2143, 2413, 2431, 3142, 3214, 3241, 3412, 3421, 4132, 4213, 4231, 4312 and 4321; and many more.

Today, we see the Catalan numbers being applied in engineering in the field of computational geometry, geographic information systems, geodesy, cryptography, and medicine. In the problems of computational geometry, they are generally used in geometric modeling. In cryptography are used in the forming of keys for secure transfer of information.

CONCLUSION

It is not exaggerated to say that the Catalan numbers are the most prominent sequence in combinatorics. They are probably the most frequently occurring combinatorial numbers after the binomial coefficients. Catalan numbers are even more fascinating than the Fibonacci numbers. Like the North Star in the evening sky, they are a beautiful and bright light in the mathematical heavens. Even today, they continue to provide a fertile ground for number theorists, especially, Catalan enthusiasts and computer scientists.

REFERENCES

[1] M. Aigner, Catalan and other numbers, in Algebraic Combinatorics and Computer Science, Springer, Berlin, 2001, 347–390.

[2] E. T. Bell, The iterated exponential integers, Annals of Math. 39 (1938), 539–557.

[3] W. G. Brown, Historical Note on a Recurrent Combinatorial Problem, Amer. Math. Monthly 72 (1965), 973–977.

[4] R. S. Calinger, Leonhard Euler: Life and Thought, in Leonhard Euler: Life, Work and Legacy (R. E. Bradley and E. Sandifer, Editors), Elsevier, Amsterdam, 2007.

[5] A. Dvoretzky and Th. Motzkin, A problem of arrangements, Duke Math. J. 14 (1947), 305–313.

[6] P. J. Larcombe and P. D. C. Wilson, On the trail of the Catalan sequence, Math. Today (1998), no. 4, 114–117.

[7] P. J. Larcombe, The 18th century Chinese discovery of the Catalan numbers, Math. Spectrum 32 (1999), no. 1, 5–7.

[8] T. Koshy, Catalan numbers with applications, Oxford Univ. Press, Oxford, 2009.

[9] J. McCammond, Non-crossing partitions in surprising locations, Amer. Math. Monthly 113 (2006), 598–610.

[10] T. P. Kirkman, On the K-Partitions of the R-Gon and R-Ace, Phil. Trans. Royal Soc. 147 (1857), 217–272.

[11] N. J. A. Sloane and S. Plouffe, The Encyclopedia of Integer Sequences, Academic Press,1995.

[12] R. P. Stanley, Enumerative Combinatorics, Vol. 2, Cambridge Univ. Press, Cambridge, 1999.

DEFINITE INTEGRALS, RIEMANN SUMS, AND AREA UNDER A CURVE

K. SOUNDARYA MOHANAN

B.Sc. Scholar, Department of Chemistry, St. Thomas College (Autonomous), Thrissur, Kerala, India

Email:soundaryamohanan@gmail.com

ABSTRACT

A clinical interviewing method was used to investigate students' understanding of elementary calculus. The analysis of responses to tasks concerned with differentiation and rate of change led to detailed data concerning the degree of understanding attained and the common errors and misconceptions. Some conclusions were drawn concerning the teaching of differentiation and rate of change.

KEYWORDS: Fundamental theorem of calculus, Riemann sums, Antiderivatives, Integrals

1. INTRODUCTION

"A teaching experiment was conducted in a calculus class to determine what it means to understand definite integrals. One interesting result was based on students' use of area under a curve as a tool for computing definite integrals. Results show that in the problems presented in this study, students' use of area under a curve was helpful in problem solving only when a deeper understanding of the structure behind the definite integral was present."

The purpose of this research was to examine student understanding of Riemann sums and definite integrals. These concepts are imperative for students to understand for three main reasons. First, many real-world applications involve functions that do not have an antiderivative that can be expressed in terms of elementary functions.

For example, the antiderivative of the function

f(x) = e^x^2

cannot be expressed in terms of elementary functions. Thus, the

Fundamental Theorem of Calculus could not be applied, and other methods for evaluating the definite integral, such as Riemann sums would be needed.

This leads to the second reason that students need to have an understanding of the structure of Riemann sums.

While Riemann sums may not be the most efficient method for

approximating a definite integral, other methods, such as the trapezoid rule, midpoint rule, or Simpson's method are based on the structure of the Riemann sum. Thus, an understanding of the structure of Riemann sums will help students to understand these other methods as well.

Setting up the appropriate definite integral requires the student to know what to integrate, and an understanding of the structure of the Riemann sum will give the student the tools he/she needs.

In all cases, it is possible to imagine the definite integral being represented by the area under a curve.

1.2. BACKGROUND

There are several pieces of literature that focus on mathematical topics that build the definite integral, Multiplication, rate of change, sequences and series, limits, and functions are all incorporated into the definite integral, and several research studies have been done to understand these topics. In addition, there are two pieces of literature that focus on the concept of integration. Orton (1983) mainly focuses on methods of evaluating definite integrals.

As is common in many calculus classes, many of the definite integral problems in Orton's study involve finding the area under the curve. He discusses the structural and calculational/executive errors that students made when finding the area under the curve in several situations.

Artigue (1991) discussed Orton's studies of calculus students' understanding of differentiation and integration.

The study found that many students could perform routine procedures for the area under a curve, but the students rarely could explain their procedures, and some even admitted that they "didn't really know why they were doing it" (Artigue, 1991).

Thompson's (1994) study focuses on student understanding of the Fundamental Theorem of Calculus. As part of his teaching experiment, he developed and implemented a module to help students understand Riemann sums in a way that develops the Fundamental Theorem. Thompson noted a distinction between accumulation and accumulating, and stressed the idea of quantities accumulating for his work.

1.3. THEORETICAL PERSPECTIVE

The theoretical perspective that has been used to analyze the data is taken from the work of Piaget (1970, 1975). The basic idea is a type of constructivism with the premise that we construct not at free will, but within certain constraints, The system in which we construct is subjected to certain laws, specifically reversibility, wholeness, transformation, and self-regulation.

Possibly the most important aspect of Piaget's constructivism, structuralism, is the concept of reflective abstraction. Abstraction "in the ordinary sense of the world" refers to something being " 'drawn out' from the things which have that property".

For example, a child learns what "red" is by seeing a lot of objects that are red. The child may be shown a red ball, a red crayon, a red shirt, and a red block and the child eventually learns the meaning of "red".

Reflective abstraction is a type of abstraction that comes from "acting on things" and ways in which we coordinate actions.

Reflective abstraction deals with the elements and the operations we perform on them. Specifically, with the definite integral, students need to do something with the components of definite integral.

Researchers have argued that the Riemann sum interpretation of the definite integral is perhaps the most valuable interpretation for making sense of integration in applied contexts, particularly in physics.

The term Riemann sum interpretation is used here to include conceptions of the definite integral as a sum of very small pieces or infinitesimals, such that the infinitesimal pieces are imagined to be a product of the value of some function and an infinitesimal change in the independent variable of the function across a selected interval of integration.

1.4. PREREQUISITES

Let us understand a few other concepts before learning about the Riemann sum.

Partition:

Suppose a closed interval $[a, b] \in R$.

The partition of this interval is said to be a sequence of the following form:

$a = x_0 < x_1 < x_2 < \ldots < x_n$

where, every $[x_i, x_{i+1}]$ is known as a sub-interval.

Norm:

The length of the biggest sub-interval is called a norm or a mesh, i.e. norm = max

Tagged Partition:

For interval [a, b], a partition P(x, t) along with a sequence of finite numbers $t_0, t_1, \ldots, t_{n-1}$ is known as tagged partition if it satisfies the condition that, $t_i \in [x_i, x_{i+1}]$ for every i.

2. RIEMANN SUMS

The Riemann sum of a real-valued function f on the interval [a, b] is defined as the sum of f with respect to the tagged partition of [a, b]. I.e.

$$\sum_{i=0}^{n-1} f(t_i)\,(x_{i+1} - x_i)\,.$$

Every term in Riemann sum denotes the area of a rectangle having length or height $f(t_i)$ and breadth x_i+1-x_i. Hence, the Riemann sum gives the area of all the rectangles and thus the area under the curve within the interval [a, b] or definite integral.

3. APPLICATION OF RIEMANN SUMS (GENERAL)

A Riemann sum is an approximation of a region's area, obtained by adding up the areas of multiple simplified slices of the region. It is applied in calculus to formalize the method of exhaustion, used to determine the area of a region. This process yields the integral, which computes the value of the area exactly.

3.1. REAL LIFE APPLICATION OF RIEMANN SUMS

- Riemann sums are the way a computer computes the integral of some numerical data, for instance the data from an accelerometer, which yields the speed of an object, and ultimately the position of the object, etc.

-Riemann sums are also used in pure mathematics to express some kind of supremum of sums as integrals.

-They can also be used in

- Integration as well as differential calculus
- They are applied from calculus to physics problems
- Used in partial differential equations and representation of functions by trigonometric series
- It is also used for the measurement of distance travelled by some object, because we can easily retrieve the average velocity of the journey and total time by the Velocity versus Time graph. The distance travelled is represented by the area under the given curve

Although having such huge applications, the Riemann integral is quite challenging to handle as we can see that its definition is little sophisticated. So, it is a bit inconvenient to use Riemann integral in practical life.

4. CONCLUSION

Integration is an easy way to find the area under a graph when you are given a function. However, when you have a set of data points that do not form an obvious equation for a function, integration becomes harder, because you would not have an equation to integrate. Therefore, in a situation where you are given a set of data points that do no the form an obvious equation for a function, it would be easier just to use Riemann sums to determine area.

By knowing the rationale behind the relationship between a definite integral and the area under a curve through Riemann sums would also help students develop their procedural knowledge by knowing what to integrate and how to set up the bounds of an integral.

However, the importance of this mathematical object has been acknowledged and has been given more prominence and identity in research in Mathematics

Thus, research conducted on the antiderivative has focused on the way students reflect on the rules of integration, its historical-epistemological meanings, and the use of technology for its introduction and study including an approach from the perspective of the theory of objectification that involves a continuous development of the meanings according to the elements used in class.

REFERENCES

[1] Artigue, M. (1991). Analysis. In D. Tall (Ed.), Advanced mathematical thinking (pp. 167-198).

[2] Boston: Kluwer.

[3] Oehrtman, M. (2002). Collapsing dimensions, physical limitation, and other student metaphors

[4] for limit concepts: An instrumentalist investigation into calculus students' spontaneous reasoning.

[5] Unpublished Doctoral Dissertation, The University ofTexas, Austin.

[6] Oehrtman, M. (2004). Approximation as a foundation for understanding limit concepts.

Paper presented at the PMENA.

[7] Orton, A. (1983). Students' understanding of integration. Educational Studies in Mathematics, 14(1), 1-18.

[8] Piaget, J. (1970). Structuralism. New York: Basic Books, Inc.

[9] Piaget, J. (1975). The equilibration of cognitive structures.

[10] The University of Chicago Press.

[11] Sfard, A. (1991). On the dual nature of mathematical conceptions:

[12] Reflections on processes and objects as different sides of the same coin.

[13] Educational Studies in Mathematics, 22, 1-36.

MATLAB APPLICATIONS IN IMAGE PROCESSING

SUJESH AREEKARA

Research Scholar, Department of Mathematics, St. Thomas' College (Autonomous), Thrissur, Kerala

Email: sujeshstc@gmail.com

ABSTRACT

In this social network-oriented century image, photographs, etc. plays a very important role. Today's present is tomorrow's past. And these images help in preserving the past. All of might have done some kind of editing in images; it can be cropping, adding filters, rotating or of some other sorts. These are possible with the aid of image processing. Digital image processing is the use of a digital computer to process digital images through an algorithm. In this paper, some basic image processing techniques using MATLAB has been discussed.

Keywords**:** Image processing, MATLAB, Face detection

Introduction

In society, the use of computers and smart phones are increasing, given the diversity of its application and the ability to store, organize and process information. We all wish to re-live our favorite moments. But it is not fully possible. Photographs represent the living memories of the past.

What is an image, exactly?!

All images, photographs, pictures, etc. we see are matrices (either a square matrix or a $m * n$ matrix). How does a matrix generate an image? How is image processing related with matrices? Here we will be discussing about both black & white and colored images.

MATLAB

What is MATLAB?

MATLAB is a high-performance language for technical computing. It integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation. Typical uses include:

- Math and computation
- Algorithm development
- Modelling, simulation and prototyping
- Data analysis, exploration and visualization
- Scientific and environmental graphics
- Application development, including Graphical User Interface building

MATLAB is the acronym for matrix laboratory. MATLAB developed by “The Mathworks Inc.” is a system of computer algebra, numerical and graphical, designed for professional use in solving problems that requires mathematical methods. In the beginning MATLAB was just a software for mathematical operations on matrix, but over the years turned into a computer system able to develop flexible essentially any technical problem. MATLAB is very easy to use.

MATLAB is an interactive system whose basic data element is an array that doesn’t require dimensioning. This allows you to solve many technical computing problems, especially those with matrix and vector formulations, in a fraction of the time it would take to write a program in a scalar non-interactive language as C.

MATLAB has evolved over a period of years with input from many users. In university environments, it is the standard instructional tool for introductory and advanced courses in mathematics, engineering, and science. In industry, MATLAB is the tool of choice for high-productivity research, development, and analysis. MATLAB users come from various background of engineering, science, and economics. MATLAB is widely used in academic and research institutions as well as industrial enterprises.

MATLAB features a family of application-specific solutions called toolboxes. Very important to most users of MATLAB, toolboxes allow you to learn and apply specialized technology. Toolboxes are comprehensive collections of MATLAB functions (m-files) that extend the MATLAB environment to solve particular classes of problems. Areas in which toolboxes are available include signal processing, control systems, neural networks, fuzzy logic, wavelets, simulation, and many others.

Components of MATLAB

The MATLAB system consists of five main parts:

- *The MATLAB language*

This is a high-level matrix/array language with control flow statements, functions, data structures, input/output, and object-oriented programming features.

- *The MATLAB working environment*

This is the set of tools and facilities that we work with as the MATLAB user or programmer. It includes facilities for managing the variables in your workspace and exporting data. It also includes tools for developing, managing, debugging, and profiling M-files, MATLAB's applications.

- *Handle graphics*

This is the MATLAB graphics system. It includes high-level commands for two-dimensional and three-dimensional data visualization, image processing, animation, and presentation graphics. It also includes low-level commands that allow you to fully customize the appearance of graphics as well as to build complete Graphical User Interfaces on your MATLAB applications.

- *The MATLAB mathematical function library*

This is a vast collection of computational algorithms ranging from elementary functions like sum, sine, cosine, and complex arithmetic, to more sophisticated functions like matrix inverse, matrix eigen values, Bessel functions, and fast Fourier transforms.

- *The MATLAB Application Program Interface (API)*

This is a library that allows you to write and Fortran programs that interact with MATLAB. It facilities for calling routines from MATLAB (dynamic linking), calling MATLAB as a computational engine, and for reading and writing MAT-files.

Image Processing

Digital image processing is the use of a digital computer to process digital images through an algorithm. All of us are familiar with *Google Lens* in Android smartphones. How does it work? Yes, you guessed it right. It uses image processing to detect and recognize the object faultlessly. Image processing has lot of applications like face detection & recognition, thumb impression, augmented reality, OCR, barcode scan and many more. There are lot of software available for image processing, among them MATLAB **i**s the most suitable to start with. MATLAB can perform many advance image processing operations, but here we wouldn’t be discussing the complex scenarios. Colours in MATLAB are coded with three numbers : the Red, Green and Blue (RGB) values.

Image Processing Using MATLAB

For a mathematician, an image is nothing but a *matrix.* If you find it hard to believe then consider the following example. Consider the matrix made up of the elements:

Fig. 1: Matrix elements of *'Felix the Cat'* image named *'STC STAM Matrix.xls'*

1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1
0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1
0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1
0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	1	1	0	0	1	1	1	1	1
0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	0	0	1	1	1	1
0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	0	0	1	1	1
0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	0	1	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	1	1
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	0	1	0	0	1
0	0	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	0	1	1	1	0	0	0	1	0	1
0	0	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	0	1	1	1	0	0	0	1	0	0
0	0	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	0	0	1	1	0	0	0	1	0	0
0	0	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	0	0	1	1	0	0	0	1	0	0
0	0	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	0	0	1	1	1	0	0	1	1	0
0	0	0	1	1	1	1	1	1	1	1	0	0	0	1	1	1	0	0	0	1	1	1	1	1	1	0
0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	0
0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	0	0	0	0	1	0	0
1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	1	0	0	0	0	0	0	0	1	0
1	1	1	1	0	0	0	1	1	1	1	1	1	1	0	0	1	1	0	0	0	0	0	0	1	0	0
1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0
1	0	0	0	1	1	1	1	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1
1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	1
1	1	1	1	1	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	1	1	1	0	1	1
1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1
0	1	1	1	1	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	1	0	0	0	1	1	1
0	0	1	1	1	1	1	0	1	1	1	1	1	1	0	1	1	1	0	0	0	0	1	1	1	1	1
1	0	0	1	1	1	1	1	0	0	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1
1	1	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1
1	1	1	1	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1
1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1

This matrix represents the image of *Felix the cat* which can be easily understood when the image is exponentially zoomed out.

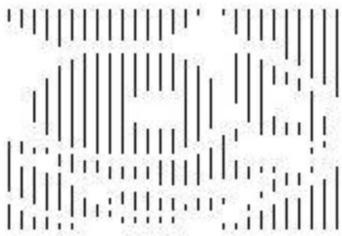


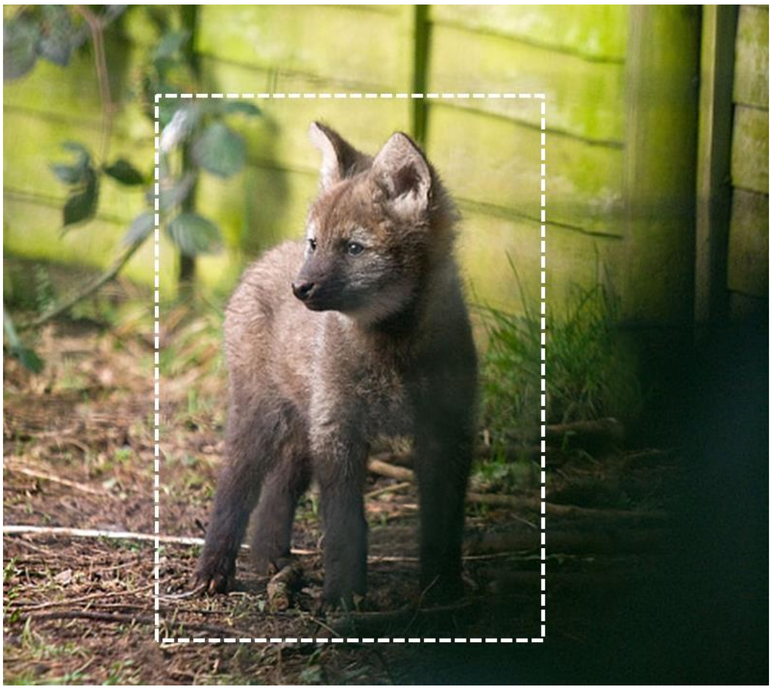
Fig. 2: Zoomed out image of aforementioned matrix

MATLAB Codes & Outputs

Felix the Cat (Black & White)	
CODE	*clc; clear all; close all;* *a=xlsread('STC ETAM Matrix.xlsx');* *imshow(a)*
OUTPUT	
Felix the Cat (Color)	

CODE	clc; clear all; close all;
	a=xlsread('STC ETAM Matrix.xlsx');
	b=56*a;
	figure(2)
	image(b)

OUTPUT	

Cropping an Image	
Ref. Image *(cropimage.jpg)*	 **Suppose we wish to crop the selected region**

CODE	clc; clear all; close all;
	a = imread('cropimage.jpg');
	b = imcrop(a,[97 60 321-97 384-60]);
	imshow(b)

OUTPUT	
Resizing an Image	

CODE	clc; clear all; close all;
	a = imread('cropimage.jpg');
	c = imresize(a,0.5);
	imshow(c)

OUTPUT	

Rotating an Image	

CODE	clc; clear all; close all;
	a = imread('cropimage.jpg');
	d = imrotate(a,75);
	imshow(d)

OUTPUT	

Creating a Collage	

CODE	clc; clear all; close all;
	a = imread('cropimage.jpg');
	b = imcrop(a,[97 60 321-97 384-60]);
	c = imresize(a,0.5);
	d = imrotate(a,75);
	subplot(2,2,1), imshow(a);
	subplot(2,2,2), imshow(b);
	subplot(2,2,3), imshow(c);
	subplot(2,2,4), imshow(d);

OUTPUT	
Enhancing an Image	

CODE	clc; clear all; close all;
	a = imread('lantern.png');
	b = imadjust(a,[0.05 0.05 0.05;1 1 1]);
	subplot(1,2,1), imshow(a);
	subplot(1,2,2), imshow(b);

OUTPUT	 Original Image	 Enhanced Image

Applying Filters

CODE

```
clc; clear all; close all;
a = imread('scene.jpg');
% a = imresize(a,0.7);
subplot(2,2,1),imshow(a);
b = rgb2gray(a);
subplot(2,2,2),imshow(b);
h = ones(100,100)/10000;
c = imfilter(b,h);
subplot(2,2,3),imshow(c);
j = ones(150,150)/22500;
d = imfilter(a,j);
subplot(2,2,4),imshow(d);
```

OUTPUT	   
Face Features Detection	
INPUT Image	
CODE	clc; clear all; close all; a = imread('APJ.jpg');

	a = imresize(a,0.3);
	
	figure(1)
	
	imshow(a)
	
	detector = vision.CascadeObjectDetector('Mouth');
	
	detector.MergeThreshold = 150;
	
	bbox = step(detector, a);
	
	videoOut = insertObjectAnnotation(a,'rectangle',bbox,'Mouth');
	
	figure(2)
	
	imshow(videoOut);
	
	detector1 = vision.CascadeObjectDetector;
	
	detector1.MergeThreshold = 25;
	
	bbox = step(detector1, a);
	
	videoOut = insertObjectAnnotation(a,'rectangle',bbox,'face');
	
	figure(3)
	
	imshow(videoOut);

OUTPUT *(Face Detection)*	
OUTPUT *(Mouth Detection)*	

Addition of Two Images	

CODE	clc; clear all; close all;
	a = imread('lantern.png');
	a = rgb2gray(a);
	b = imread('scene.jpg');
	b = rgb2gray(b);
	a = imadjust(a,[0.1 0.1 0.1;1 1 1]);
	c = imresize(b,[size(a,1) size(a,2)]);
	d = a+c;
	imshow(d)

OUTPUT *(Black & White)*	

OUTPUT *(Color)*	

Concluding Remarks

The implications of MATLAB as a tool for image processing have been discussed. Some further complex applications where MATLAB can be used as an image processing tool are:

- Face Features Detection
- Color Thresholding
- Edge Detection
- OCR (optical character recognition) in Texts & Natural Images
- Detecting Cars in Traffic videos
- Identifying Round Objects

References

[1] Wikipedia.
[2] A. K. Jain, "Fundamentals of Digital Image Processing" (1989).

[3] Gomes J, Velho L; Image processing for Computer Graphics and Vision, Springer-Verlag (2008).

[4] Mathworks. Using MATLAB, version 6.5. the Math Works, Inc. Natick MA. 2002.

[5] Pratt, W.K. Digital image processing. John Willey & sons, InC., third edition, 2001.

NANOFLUIDS AND APPLICATIONS

SANILA MARY DAVIS

MSc Scholar, Department of Mathematics, St. Thomas' College (Autonomous), Thrissur, Kerala.

Julie Andrews

Assistant Professor, Department of Mathematics, St. Thomas' College (Autonomous), Thrissur, Kerala.

ABSTRACT

In nature there are different types of fluids and they are basically use as the heat carriers in the heat transfer applications. But the main limitation of the fluid is the low thermal conductivity. And in 1995, Choi developed a newly innovative class of heat transfer fluids and call it nanofluids. Because of the thermal, physical properties and heat transfer performance of nanofluids, they receiving a great attention now a days. And this paper mainly showcases the cooling applications of nanofluids.

1.INTRODUCTION

Any substance which has the property of flowing is defined as a fluid .Nano particles are the particles which has the size smaller than 100nm.And nanofluids are the fluids consists of nanoparticles and occur by suspending nanoparticles in a fluid medium such as water ,oil and ethylene glycol etc. The nanoparticles which we used in the nanofluids are most commonly made up of metals, oxides, carbides, or carbon nanotubes. However low thermal conductivity is an important limitation of fluids and nanofluids can overcome this important limitation. Because nanofluids has great thermal, physical properties such as thermal conductivity, thermal diffusivity, viscosity and heat transfer coefficient compared to these base fluids. And the most key feature of the nanofluids is its super thermal conductivity. And because of this feature this goes long way in increasing the rate of heat transfer. And by varying size of the particles the conductivity of nanofluids can be controlled. Only by the chemical solution method the production of efficient nanofluids with controllable micro structures possible. Because of the chemical solution method, it enables us to designed out microstructures that will helps us to control the chemical reactions which will takes place at a rapid time and will gave a high degree of precision. And there are two of type preparation methods for nanofluids one is the Two-step method and the other one is the One–step method. In Two-step method the nanoparticles, or nanofibers or nanomaterials which we used here as dry powders were produced by physical or chemical process. And after that directly dispersed these nanosized powders into the fluids with the help of intensive magnetic force agitation, ultrasonic agitation. And this is the method which use most commonly for the preparation of nanofluids. And it is the most economic method to produce nanofluids as large scale. And also, there exist difficulty in preparing stable nanofluids by Two-step method then to overcome this problem there introduces another technology call One–step method.

Making and dispersing the particles simultaneously is the process of one-step method. In this method agglomeration of nanoparticles is minimized by avoiding the drying, storage, transportation and dispersion of nanoparticles are the stability of the nanoparticles increased.

2.APPLICATIONS

Now a days most of the industrial process contains transfer of heat energy. And also, the most major task of the industrial necessity is the process of removing, adding heat to the industrial facility or to move from one steam of process to another. Primarily nano fluids are used for the enhanced thermal properties as coolants in the heat transfer equipment’s like electronic cooling systems and radiators.

1.1. COOLING APPLICATIONS

There are so many cooling applications for nanofluids. Crystal silicon mirror cooling, Electronics cooling, Vehicle cooling, Transformer cooling, Defense application are some of the applications of nanofluids.

2.1.1. CRYSTAL SILICON MIRROR COOLING

We use crystal silicon mirrors in the high intensity X-ray sources. And one of the most important applications of nanofluids is the developing an advanced cooling technology to cool these crystal silicon mirrors.LEE and CHOI carried out an observation and find out the performance of microchannel heat exchangers with water, liquid nitrogen and nanofluids as the working fluid. And after by comparing the performance of nano fluid cooled microchannel heat exchanger with water cooled and liquid nitrogen cooled microchannel heat exchanger the result shows that nanofluids can widely reduce the thermal resistances and increase the power densities. LEE and CHOI who introduce advanced cooling technology develop the concept of microchannels filled with nanofluids. And this advanced cooling technology has more efficient cooling than that of other cooling technology. Because the microchannel use here increases the effective heat transfer area and also the metallic nanoparticles increase the effective thermal conductivity of coolants.

2.1.2. ELECTRONICS COOLING

It was Chine et al who first shows experimentally the thermal performance of heat pipes can enhance by the use of nanofluids .For that they used water based nanofluids which contains17-nm gold nanoparticles as the working fluid in a disk shaped miniature heat pipe(DMHP).And after then measures the thermal resistance of DMHP with nanofluids and also measures the thermal resistance of DMHP with deionized water(DI).And the result shows that when we used nanofluids instead of (DI)water thermal resistance will reduced.

2.1.3. VEHICLE COOLING

Nano particles can disperse in coolants and engine oil, but nanoparticles can also disperse into the transmission fluids, gear oils, and other fluids and lubricants. Thus, the nanofluids which occur by dispersing nanoparticles into the transmission fluids, gear oils, and other fluids and lubricants provides better overall thermal energy and lubrication. It was Tzeng et al who probably first apply the nanofluid research in cooling in real world automatic power transmission system. To investigate the optimum possible compositions of nanofluids for higher heat transfer performance they dispersed nanoparticles of copper oxide and aluminium oxide into the automatic power transmission oil. They used real rotary blade coupling (RBC) of a power transmission system of a real-time four-wheel drive vehicle as the platform for their experiment. The designing of RBC must be so precise because if the local temperature is higher than the 266°F, the excessive thermal stress occur it will damage its roaring components. As a result, power cannot be transmitted to the real wheels which affected the vehicle performance severely. And also, if RBC becomes damaged, they can’t be repairable but should replacable. Therefore, it is important that to improve the heat transfer efficiency to contain the excessive thermal stress on the components of the power transmission system. Simulating the conditions of a real car at various rotating speed they measured the temperature distribution of the RBC at four different rotating speeds (400,800,1200,1600rpm). And at the both high and low rotating speeds the results shows that the CuO nanofluids have the lowest temperature distribution. And this is the best heat transfer effect. This work is important because it is the real-world application of nanofluids. And this makes a big step forward for the industrial applications of nanofluids.

2.1.4. TRANSFORMER COOLING

Transformer cooling is an application of nanofluids for reducing the size and weight of the transformer, because of this reason power generation industries are interested. It was Xuan and Li and Yu et al who demonstrated that by using of nanoparticles additives the heat transfer properties of transformer oil can improved. By adding nanoparticle additives to transformer oils results in either a reduction in size of new transformers at the same level of power transmitted or an increase in the performance of existing transformers. Specifically, we can say cooling fluids in transformers of next generation will be nanofluid based transformer oil

2.1.5. DEFENSE APPLICATION

There are number of military devices and systems, such as military electronics, military vehicle components, radars and lasers which need high heat flu cooling. And using the conventional heat transfer fluid for the cooling is so difficult. Because of the limited capability of current heat transfer fluids cooling of direct energy weapon and associated power electronics are critical. But nanofluids can overcome this problem. Thus nanofluids provides cooling technology for defense applications

3.CONCLUSION

Nanofluids has various exciting applications in science and technology. Because of the small size of the particles, it can apply in various places. Only by the help of future research the industrial production of nanofluids will happen. Nanofluids research give way to developing next generation coolants for numerous engineering applications. This paper presented a review of cooling applications of nanofluids.

REFERENCES

[1] Jang S P & Choi S U S, 2004.Role of Brownian motion in the enhanced thermal conductivity of nanofluids, Appl.phys. Lett.84,4316-4318.

[2] XuanY &Li Q ,2003, Investigation on convectiveheat transfer and flow features of

nanofluids,J. Heat Transf.,125,151-155.

[3] T.P.Otanicar ,P.E.Phelan,R.S.Prasher ,G.Rosengarten,R.A. Taylor,2010,Nanofluidbased direct absorption solar collecter,J.Renew.Sustain.Energy 2,033102.

[4] H. Tyagi, P. Phelan, R. Prasher,2009, predicted efficiency of a low –temprature nanofluids- based direct absorption solar collector, J. Solar Energy Eng.131,041004.

[5] OmidMahian a,Ali Kianifar a, Soteris A.Kalogirou b, loan Pop c,SomchaiWongwises,2013,A review of the applications of nanofluids in solar energy,International Journal of Heat and Mass Transfer,57,582-594

[6] Hofmann W-K, de Vos S, Komor M, Hoelzer D, Wachsman W and Koeffler H P,2002, characterization of gene expression of CD34+ cells from normal and myelodysplastic bone marrow Blood 100 3553-60.

[7] Seesod N, Nopparat P, Hedrum A, Holder A, Thaithong S, Uhlen M and Lundeberg j,1997, An integrated system using immunomagnetic separation, polymerase chain reaction, and colorimetric detection for diagnosis of plasmodium falciparm Am.J.Tropical Med.Hygiene 56 322-8

[8] Paul F, Melville D,Roath S and Warhurst D,1981,A bench top magnetic separator for magnetic separator for malarial paracite concentration IEEE Trans.Magn.MAG-17 2822-4

[9] Liberti P A,Rao C G and TerstappenLWMM 2001 Optimization of ferrofluids and protocols for the enrichment of breast tumour cells in blood J.Magn.Magn.Master.225 301-7

[10] Pak B & Cho Y I, 1998, hydrodynamic and heat transfer study of dispersed fluid with submicron metallic oxide particle, Exp. Heat Transfer,11,151-170.

ROLE OF MATHEMATICS TOOLS IN BUSINESS DECISION MAKING

LINNI WILSON

Assistant professor, Department of Commerce, St. Thomas College Autonomous Thrissur

Email: linniwilson@gmail.com

ABSTRACT

Mathematics is a science of numbers, symbols and diagrams. Mathematical tools hold an important role in every fields like Economics, Commerce, and Industry etc. Mathematical tools are used in business economics in estimating various economic relationship and employing them in business decision making and forward planning. Business decisions uses considerable amount of Algebra, Geometry, Logarithms, Calculus etc. Business decisions are becoming more and more mathematical today. In this purview this paper attempts to analyses the role of mathematical tools in decision making process to achieve business objective.

KEYWORDS

Decision making, Business Mathematics, Linear programming, Operation Research

INTRODUCTION

Decision making is a necessary aspect of daily life. In every fields, in order to achieve objective good decision-making process is required. Decision making is the process of selecting the best alternative from the available alternatives. It’s not easy to take decisions, we need to apply scientific tools to derive best result. Scientific decision-making process involves the use of mathematical tools like linear programming, algebra, probability etc. Business organizations are using mathematical tools in the areas of forecasting, production, sales, inventory management, accounting, and financial analysis etc. In every aspect of business, one feels helpless without mathematics. Because business deals with monetary transactions and this money element makes it extremely to have sufficient knowledge about the basics of mathematical calculations. This study tries to analyze the application of mathematical tools in business decision making process.

OBJECTIVES

- To analyze the role of mathematical tools in business decision making process

RESEARCH METHODOLOGY

It is a descriptive research based on secondary data

ANALYSIS

Decision Making Process

Decision making is a key part of a manager's activity. According to Peter Drucker "Whatever a manager does, he does through decision making". Decision making is not a separate function of management. Rather, it is present in every managerial function like planning, organisation, direction and control. Decision making is the selection of best possible alternative from among the various alternatives for the solution of a given problem.

Steps in Decision Making Process

- Define the problem or opportunity
- Determine objectives
- Develop alternatives
- Evaluate alternatives
- Choose the best alternative
- Implement the chosen alternative
- Follow up

Application of Mathematics in Business Decision making process

The stream Business Mathematics reveals the basic applications of mathematics in business firm. Business involves the buying and selling of goods in order to earn profit, it uses mathematics to record, classify, summarize and analyze the business transactions. So mathematics is used by commercial enterprises to record and manage the business operations such as, elementary arithmetic involving fractions, decimals, percentage, elementary algebra, statistics and probability. Now a day's business management is using advanced mathematics such as calculus matrix algebra and liner programming. Practical applications include checking accounts, forecasting the sales, price discounts, mark-ups, mark-downs, payroll calculations, simple and compound interest, reducing wastage of resources. Some applications of mathematics in business and commerce are listed below:

Matrix

Matrix is an arrangement of numbers into rows and columns. It is a two-dimensional arrangement of numbers. Linear equations can be analyzed through matrix. Commercial organizations use Matrix for developing solutions. It gives opportunities to finance and logistics management and customer relationship by providing them a variety of solutions. As total cost, revenue, supply, demand and population are all related with a system of linear equations. The Leontief model production equation in input-output analysis uses matrix to Predict what happens to an economy when final demand changes. By changing the consumption matrix this can represent what happens to an economy when the relative cost in terms of other goods (a change in one or more entries in internal demand) of producing one good can change both internal and final demand economy.

When you are faced with multiple choices and several variables, a decision matrix can clear up confusion and highlight considerations that may factor into the final outcome. This quantitative method can remove emotion as well as confusion to help you lead your business to success.

Unlike a simple list of pros and cons, a decision matrix allows you to place importance on each factor and weigh them accordingly

Operation research

Operation Research is the application of scientific methods, techniques and tools to problems involving the operations of a system so as to provide those in the control of the system with optimum solutions to the problems. Operations research (OR) is an analytical, logical and systematic method of problem-solving and decision-making that is helpful in the management of organizations. In operations research, problems are broken down into basic components and then solved in defined steps by mathematical analysis. Analytical methods used in Operation research include mathematical logic, simulation, network analysis, queuing theory and game theory etc. Business organizations uses techniques of operation research like Inventory Control Models, Waiting Line Models, Replacement Models, Allocation Models, and Linear programming Technique etc. to derive solutions for various problems relating to stock keeping, maintenance, transportation, assignments and product mix selection.

In **transportation decision** various centers with their demands are given and various warehouses with their stock positions are also known, then by using linear programming technique, we can find out most economical distribution of the products to various centers from various warehouses.

In **product mix selection** decision operation research techniques can be applied to determine best mix of the products for a plant with available resources, so as to get maximum profit or minimum cost of production.

The technique of operation research can be applied for **assignment of different personnel** with different aptitude to different jobs so as to complete the task within a minimum time.

Many **financial decision-making** problems can be solved by using linear programming technique.

Algebra

Algebra is a branch of mathematics dealing with symbols and the rules for manipulating those symbols. In elementary algebra, those symbols represent quantities without fixed values, known as variables. Mathematical principles are needed to study accounting. It incorporates successful exploration of numerical, geometrical and logical relationships. Mathematics benefits accountant in comparison – mathematical formulas help business and commerce to compare income, cost, expenses and profits. The various formulas are derived using various percentage, ratios and equations. The various ratios are derived such as: inventory turnover ratios, profitability ratios, debtor turnover ratio, debt-equity ratio etc. Mathematics is helpful in deriving accounting equation. The basic idea in accounting is that total wealth of business is called Assets. There are two possible claims on assets (A) called liabilities (L) and capital(C). By using mathematical relation, A=L+C, accountants use mathematics in order to arrive the total cost and taking decision regarding manufacturing or buying the product. The total cost formula for business is T= a+bx; where "T" is total cost, "a" is fixed cost, "b" is cost per unit produced and "x" is no. of units produced. Also, profits are determined by subtracting total cost from total revenue and helps in analyzing the financial health of business and prices are determined by adding some mark-up to cost. So, accountant used addition and percentage to determine the prices of product

Calculus

Calculus is the study of how things change. It provides a framework for modeling systems in which there is change, and a way to deduce the predictions of such models. In business we come across many such variables where one variable is a function of the other. For example, the quantity demanded can be said to be a function of price. Supply and price or cost and quantity demanded are some other such variables. Calculus helps us in finding the rate at which one such quantity changes with respect to the other. Marginal analysis in Economics and Commerce is the most direct application of differential calculus. In this context, differential calculus also helps in solving problems of finding maximum profit or minimum cost etc., while integral calculus is used to find he cost function when the marginal cost is given and to find total revenue when marginal revenue is given.

Probability

Statistics is very indispensable for the businessman. It formulates various plans and policies and forecasts trends of future such as change in demand, market fluctuations using statistical techniques. On the other hand, future events are uncertain and to predict these uncertainties, probability is an effective tool to forecast sales, scenario, future returns and risk evaluation in the business world. Before introducing the product, team of market research analyze data relating to population, income of consumer, tastes, preferences, habits, pricing policy of competitors by using various statistical techniques. We can collect and analysis the data in the field of economy by using statistical methods. Probability theory serves as a useful tool for decision making, estimating number of defective units, sales expected and also in business policies. The application of probability theory in small business was examined to find the implications and in restoring the gap between the rich and the poor through better and informed decisions. The probability theory has wide application in small business firms; probability shows specificity in business situations and is inevitable in this era of information overload caused by ICT. In nutshell, statistics and probability are very useful in taking various decisions relating to material, production, finance, personnel and marketing in an Industry.

Using statistical data for analyzing various business activities have several benefits:

- It simplifies the decision-making process by neglecting complex and confusing situations.
- It helps in creating a decision model.
- The testing and interpretation of the decision become easier.
- The data driven by using business mathematics can be used again and again for similar situations.
- Modification becomes easier

CONCLUSION

The mathematical methods and tools become crucial part of the business organization. Application of mathematics becomes necessary from business decision making. Mathematical tools application is evident from buying or estimating the cost of product to the end sales and earning profits. Mathematical formulae help business to do financial analysis using ratios, percentages, equations. The objective of minimizing cost & maximizing profit is achieved through linear programming and calculus. The estimation of future returns & profitability is done through probability distributions. It also helps in sale forecasting & risk evaluation. Matrices play important role in variety of solutions for consumer relationships and logistics management. Statistics helps in collection, presentation and analysis of data to arrive at conclusions. So, we can conclude that mathematical tools play an important role in business decision making.

REFERENCES

[1] Abraham, B. (2007) Implementation of Statistics in Business and Industry. Colombian Journal of Statistics,

[2] Herrmann, R.A. (2007). Modern mathematics: Its relation to physical science and theology. Mathematics Department, United States Naval Academy. Retrieved from

http://www.serve.com/herrmann/math.htm.

[3] Veer, D., Shukla. P., Mathematics (2009) - "The Queen of Commerce", Shodh, Samiksha aur Mulyankan, 2(6)

[4] Kaur, Paul Satinder (2017) Role of Mathematics in Business and Commerce, Retrieved from http://www.conferenceworld.in

Fuzzy Riesz $lG-$ submodule

Sowmya K[a,*], Sr. Magie Jose[b]

[a]Assistant Professor,St.Mary's College,Thrissur,Kerala,India. Pin:680020.
[b]Associate Professor,Department of Mathematics, St.Mary's College,Thrissur, Kerala, India. Pin:680020.

Abstract

A fuzzy *Riesz* $lG-$ *submodule* of a *Riesz* $lG-$ *module* is defined and properties are listed. A convex fuzzy *Riesz* $lG-$ *submodule* is also defined and properties are studied.

Keywords: fuzzy Riesz $lG-$ *submodule ;convex fuzzy Riesz* $lG-$ *submodule;Riesz* $lG-$ *module; Riesz space*.

1. Introduction

Fuzzification of crisp algebraic structures leads to structures having many practical applications. Fuzzy vector spaces are studied in[3]. Fuzzy vector lattices are discussed in[5, 4].Fuzzy G-Modules is introduced in[6]. Fuzzy lattice ordered G-modules is studied in [10]. In this paper, a fuzzy *Riesz* $lG-$ *submodule* is introduced based on *Riesz* $lG-$ *module* in [7].

2. Preliminaries

In this section, some basic definitions and results are reviewed.

Definition 2.1. [8] Let L be a poset and $x,\ y \in L$ are such that $x \leq y$, then an *interval* is defined by $[x, y] = \{z \in L : x \leq z \leq y\}$. A non-empty subset C of a poset L is said to be *convex* if $[a, b] \subseteq C$ for all $a,\ b \in C$ with $a \leq b$.

Definition 2.2. [8] A *Lattice* L is a poset in which the infimum $x \wedge y$ and supremum $x \vee y$ exist for any pair of elements x and y in L.

Definition 2.3. [8] A non empty subset M of a lattice L is said to be a *sublattice* of L if $x \wedge y$ and $x \vee y \in M$ for all $x,\ y \in M$.

Definition 2.4. [1] Let G be a group and $\leq$ be a partial order on it. Then G is a *lattice ordered group* or an *l-group* if

1. $(G,\ \leq)$ is a lattice.
2. $g_1 \leq g_2 \implies x * g_1 * y \leq x * g_2 * y$ for all $x, y, g_1, g_2 \in G$.

Definition 2.5. [1] Let G be an l- group. The *positive cone* of G is the set $G^+ = \{g \in G : g \geq e\}$ whose elements are termed as positive elements of G and the *negative cone* of G is the set $G^- = \{g \in G : g \leq e\}$ which contains all negative elements of G.

Definition 2.6. [1] Let G is a l- group. Then for $g \in G$ the *positive part* of g is $g^+ = g \vee e \in G^+$, and the *negative part* is $g^- = g^{-1} \vee e \in G^+$. The *absolute value* of g is $|\,g\,| = g \vee g^{-1} = g^+ * g^-$ and $|\,g\,| \in G^+$.

Theorem 2.7. [1] Let G be an l-group and $P = G^+$ be the positive cone. Then for all $g \in G,\ g\,Pg^{-1} = P$.

Definition 2.8. [2] A real vector space V is an *ordered vector space* if it satisfies the following conditions

1. $x \leq y$, then $x + z \leq y + z$
2. $x \leq y$, then $\lambda\, x \leq \lambda\, y$ for all $x,\ y,\ z \in V$ and $0 \leq \lambda \in \mathbf{R}$.

Definition 2.9. [2] A *vector lattice* or a *Riesz space* is an ordered vector space which is also a lattice.

Example 2.10. [2] The Euclidean space $\mathbf{R}^\mathbf{n}$ is an example for a vector lattice (Riesz space) under the *product order* given by, $x \leq y$ if $x_i \leq y_i$ for all $i = 1, 2, \ldots, n$. The supremum and infimum of two elements x and y is defined as $x \vee y = x_i \vee y_i$ and $x \wedge y = x_i \wedge y_i$ for all $i = 1, 2, \ldots, n$.

Definition 2.11. [2] A *vector sublattice* *(Riesz subspace)* of a vector lattice (Riesz space) is a vector subspace which is also a sublattice.

Definition 2.12. [7] Let G be an l-group. A Riesz space E is called a *Riesz* $lG-$ *module* if the group action G on E denoted by $g \circ x \in E$ for all $g \in G$ and $x \in E$ and has the following properties

(i) : $e \circ x = x$

(ii) : $(g * h) \circ x = g \circ (h \circ x)$

(iii) : $g \circ (rx + sy) = r(g \circ x) + s(g \circ y)$

(iv) : $|g| \circ (x \wedge y) = (|g| \circ x) \wedge (|g| \circ y)$

$|g| \circ (x \vee y) = (|g| \circ x) \vee (|g| \circ y)$

$(g \wedge h) \circ |x| = (g \circ |x|) \wedge (h \circ |x|)$

$(g \vee h) \circ |x| = (g \circ |x|) \vee (h \circ |x|)$ for all $g, h \in G$, $x, y \in E$, $r, s \in \mathbf{R}$.

Remark 2.13. [7] $g \circ 0 = 0$ for all $g \in G$.

Example 2.14. [7] $\mathbf{R}^2$ is a *Riesz* $lG-$ *module* under the action of $\mathbf{R}^+$, the set of positive real numbers, where the group action is defined by $r \circ (x, y) = (rx, ry)$, for $r \in \mathbf{R}^+$ and $(x, y) \in \mathbf{R}^2$.

Definition 2.15. [7] Let E be a *Riesz* $lG-$ *module*. A vector sublattice (Riesz subspace) F of E is a *Riesz* $lG-$ *submodule* or $RlG-$*submodule* of E if F itself is a *Riesz* $lG-$ *module* under the same action of G as that on E.

3. Fuzzy *Riesz* $lG-$ *submodule*

Definition 3.1. A fuzzy subset ϱ of a *Riesz* $lG-$ *module* E is called a fuzzy *Riesz* $lG-$ *submodule* or a fuzzy $RlG-$submodule if

(i) : $\varrho(rx+sy) \geq \varrho(x) \wedge \varrho(y)$

(ii) : $\varrho(x \wedge y) \geq \varrho(x) \wedge \varrho(y)$

(iii) : $\varrho(x \vee y) \geq \varrho(x) \wedge \varrho(y)$

(iv) : $\varrho(g \circ x) \geq \varrho(x)$ for all $g \in G,\ x, y \in E,\ r, s \in \mathbf{R}$.

Example 3.2. The euclidean plane $\mathbf{R^2}$ is a *Riesz* $lG-$ *module*. If we define a fuzzy set ϱ in $\mathbf{R^2}$ as

$$\varrho(x_1, x_2) = \begin{cases} 1, & \text{if } x_1 = x_2 \\ 0, & \text{otherwise} \end{cases}$$

then ϱ is a fuzzy *Riesz* $lG-$ *submodule*.

Theorem 3.3. *Let* ρ *be a fuzzy Riesz* $lG-$ *submodule of a Riesz* $lG-$ *module* E. *Then for any* $x \in E$,

(i): $\varrho(0) \geq \varrho(x)$

(ii): $\varrho(x^+) \geq \varrho(x)$

(iii): $\varrho(x^-) \geq \varrho(x)$

(iv): $\varrho(| x |) \geq \varrho(x)$

Theorem 3.4. *Let* ϱ *be a fuzzy* $RlG-$ *submodule of a Riesz* $lG-$ *module* E. *Then for* $\alpha \in (0, 1]$, *the level set* ϱ_α *defined by* $\varrho_\alpha(x) = \{x \in E : \varrho(x) \geq \alpha\}$ *is a* $RlG-$ *submodule of* E *if and only if* ϱ *is a fuzzy* $RlG-$ *submodule of* E.

Theorem 3.5. *Let* E *be a Riesz* $lG-$ *module. Then a nonempty subset* F *is a* $RlG-$*submodule of* E *if and only if the characteristic function of* F *is a fuzzy* $RlG-$ *submodule of* E.

Theorem 3.6. *Let* E *be a Riesz* $lG-$ *module and* ϱ *be a fuzzy* $RlG-$ *submodule of* E. *For* $k \in \mathbf{R}^+$, *the fuzzy set* $k\varrho$ *defined by,* $k\varrho(x) = \varrho(kx)$ *is a fuzzy* $RlG-$ *submodule of* E.

Proof. Let $x, y \in E$, $g \in G$ and $r, s, \in \mathbf{R}$.

$(k\varrho)(rx + sy) = \varrho(k(rx + sy)) = \varrho(k(rx) + k(sy)) = \varrho(r(kx) + s(ky)) \geq \varrho(kx) \wedge \varrho(ky) = (k\varrho)(x) \wedge (k\varrho)(y)$.

$(k\varrho)(x \wedge y) = \varrho(k(x \wedge y)) = \varrho(kx \wedge ky) \geq \varrho(kx) \wedge \varrho(ky) = (k\varrho)(x) \wedge (k\varrho(y))$.

Similarly, we can prove that $(k\varrho)(x \vee y) = (k\varrho)(x) \wedge (k\varrho(y))$.

Also,$(k\varrho)(g \circ x) = \varrho(k(g \circ x)) = \varrho(g \circ (kx)) \geq \varrho(kx) = (k\varrho)(x)$. □

Definition 3.7. *A fuzzy subset ϱ of a Riesz $lG-$ module E is called a convex fuzzy Riesz $lG-$ submodule or convex fuzzy $RlG-$submodule of E if $\varrho(z) \geq \varrho(x) \wedge \varrho(y)$ for all $x, y \in E$ with $x \leq z \leq y$.*

Theorem 3.8. *Let ϱ be a fuzzy $RlG-$ submodule of a Riesz $lG-$ module E. Then for $\alpha \in (0, 1]$, the level set ϱ_α defined by $\varrho_\alpha(x) = \{x \in E : \varrho(x) \geq \alpha\}$ is a convex $RlG-$ submodule of E if and only if ϱ is a convex fuzzy $RlG-$ submodule of E.*

Proof. Let ϱ is a convex fuzzy $RlG-$ submodule of E and $x, y \in \varrho_\alpha(x)$ with $x \leq z \leq y$. Then $\varrho(z) \geq \varrho(x) \wedge \varrho(y) \geq \alpha$. Hence, $z \in \varrho_\alpha(x)$,which proves the one part of the theorem. Conversely, let $x, y \in \varrho_\alpha(x)$, $x \leq z \leq y$ then $z \in \varrho_\alpha(x)$.Then $\varrho(x) \leq \varrho(z) \leq \varrho(y)$.Therefore ,$\varrho(z) \geq \varrho(x) \geq \varrho(x) \wedge \varrho(y)$ and ϱ is a convex fuzzy $RlG-$ submodule of E. □

4. Conclusion

In this paper, a fuzzy *Riesz $lG-$ submodule* and a convex fuzzy *Riesz $lG-$ submodule* of a *Riesz $lG-$ module* are defined and some basic properties are listed.The idea can be extended to further research in this field.

References

[1] T.S Blyth, *Lattices and Ordered Algebraic Structures*, Springer-Verlag, (2005).

[2] R. F. Gloden, *Lectures on Riesz given by Prof. A. C. Zaanen*, EU publications, (1966).

[3] A. K Katsaras, D. B Liu, Fuzzy Vector Spaces and Fuzzy Topological Vector Spaces, *Journal of Mathematical Analysis and Applications*, 58,(1977) 135-146.

[4] C. C. A Labuschagne, A. L Pinchuck, The set of fuzzy points of a fuzzy vector lattice is not a vector lattice, *Fuzzy Sets and Systems*,157,(2006) 2783 -2785.

[5] M.S. Samuel, Fuzzy vector lattices, *J. Fuzzy Math.* 3, (1995) 405-409.

[6] Shery Fernandez, Fuzzy G-modules and fuzzy representation, TAJOPAAM, I(2002) 107-114.

[7] K. Sowmya, J. Magie, P. Ursala, Introduction to *Riesz* $lG-$ *module*, *Malaya Journal of Matematik*, 8(2) (2020), 561-564.

[8] S.A Steinberg, *Lattice-ordered Rings and Modules*, Springer (2010).

[9] P. Ursala, P. Isaac, An Introduction to Lattice Ordered G-Modules, *Journal of Global Research in Mathematical Archives*, Volume 4, No.11, (2017) 76- 82.

[10] P. Ursala, P. Isaac, Fuzzy Lattice Ordered G-modules, *International Journal of Fuzzy System Applications*, Volume 8, Issue 3, (2019).

A REVIEW ON BOUNDARY-LAYER FLOW OF A NANOFLUID PAST A STRETCHING SHEET

B. SARAVANAN,

Research Scholar Department of Mathematics, St. Thomas College (Autonomous), Thrissur.

Email: saravananbs82@gmail.com

ABSTRACT

The focal aim of presentation is to review the paper titled" Boundary-layer flow of a nanofluid past a stretching sheet" authored by W.A. KHAN and I. POP.The problem of laminar fluid flow which results from the stretching of a flat surface in a nanofluid has been investigated numerically. This is the first paper on stretching sheet in nanofluids. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. A similarity solution is presented which depends on the Prandtl number Pr, Lewis number Le, Brownian motion number Nb and thermophoresis number Nt. The variation of the reduced Nusselt and reduced Sherwood numbers with Nb and Nt for various values of Pr and Le is presented in tabular and graphical forms. It was found that the reduced Nusselt number is a decreasing function of each dimensionless number, while the reduced Sherwood number is an increasing function of higher Pr and a decreasing function of lower Pr number for each Le, Nb and Nt numbers.

REFERENCES

[1] H.S. Takhar, A.J. Chamkha, G. Nath, Unsteady three-dimensional MHDboundary-layer flow due to the impulsive motion of a stretching surface,Acta Mech. 146 (2001) 59–71.

[2] J. Vleggaar, Laminar boundary layer behaviour on continuous accelerating surface, Chem. Eng. Sci. 32 (1977) 1517–1525.

[3] L.J. Crane, Flow past a stretching plate, J. Appl. Math. Phys. (ZAMP) 21 (1970) 645–647.

[4] K.N. Lakshmisha, S. Venkateswaran, G. Nath, Three-dimensional unsteady flow with heat and mass transfer over a continuous stretching surface, ASME J. Heat Transfer 110 (1988) 590–595.

Emerging Role of Cloud Encryption among Corporates- A Review

Ms. Sneha Silvester (Asst.professor, Department of Commerce, St.Thomas college (Autonomous) Thrissur.

Ms Alice N K (Asst.professor, Department of Management Studies, St. Thomas College (Autonomous) Thrissur.

Abstract

Cloud encryption is the process of encoding or transforming data before transferring to cloud storage in order to conceal from unauthorized users. Corporate data contains sensitive information. Mishandling information results in brand damage. Cloud encryption provide protection to confidential information of organizations transmitted through the internet. This paper aims to study various business problems solved by adopting cloud encryption.

Keywords: Encryption, Corporates

1.Introduction

Online data security has been one of the prominent issues faced by corporates today. The unstoppable transfer of data from analog to digital devices and applications means our data is more susceptible to risks and vulnerabilities. One of the most effective tools to protect our data is cloud encryption. Using the cloud encryption anyone is able to log in anywhere with Internet access to upload and retrieve their data.

Encryption is defined as, “the translation of data into a secret code." Cloud encryption is the process of encoding or transforming data before it’s transferred to cloud storage. Encryption uses mathematical algorithms to transform data to an unreadable form that can conceal it from unauthorized and malicious users.

The various methods used for cloud encryption are symmetric algorithm and asymmetric algorithm. In symmetric algorithm, encryption and decryption keys are the same, which makes it best for closed systems and individual users. These keys are used to secure communication. This is also known as the secret key algorithm and is usually used for bulk data encryption. In asymmetric algorithm two keys are used (private and public) and they are mathematically linked together. The private key must be kept hidden and secret, but the public key can be shared with anyone.

Cloud encryption is needed because its main aim is to secure and protect confidential information as it is transmitted through the Internet and other computer systems.

Information technology mainly focuses on the availability of the data and its integrity. IT does not give enough thought on data confidentiality. This is why cloud encryption should be used by any organization.

2.Objective of the Study

The main objective of this study is to find out the major business problems to be solved with the help of implementing cloud encryption.

3. Analysis and Interpretation

Encryption is an effective method of protecting your corporate data, in the same way locking the doors to your business is an effective method of preventing trespassers. The hacking and selling of corporate data can be a very lucrative prospect for a potential hacker and, as such, protection against hacking is extremely important.

Leaving company data unencrypted on the company network is the equivalent of leaving your business' doors unlocked and all of the cash from the week in the register. Once hackers learn that your data is unencrypted, it becomes an easy target.

Comparing hacking (virtual theft) with burglary (physical theft), there is a much higher chance that your business will be hacked than there is that your business will be broken into. According to a recent survey, 90% of businesses say they have been hacked. Burglary statistics vary by region, but are typically extremely low (well under 1%).

The initial damage related to a burglary is obvious, but the damage related to the theft of sensitive company data, such as client lists or payroll information, could be far more deadly than a few thousand dollars in damages from a burglary.

.Major Business **problems** solved by Adopting Data Encryption

1**. Importance of data and where it is stored**

Your business should have a clear concept of the value (and sensitive nature) of the data that is critical for operations. The inherent need to undertake efforts to assess risks and costs involved with current data storage practices is real. Especially in an international business organization deciding where to house data is a complex question that is largely determined by how that data will be utilized.

Many CIOs prefer to keep their companies data relatively nearby, and some of them will only work with companies that house data domestically. That is often difficult for large companies with offices in multiple locations, so it's important to look at what you're using your data for to decide where it should (legally) be stored.

Business have access to more data than ever, but storing it can be tricky. While some business choose to only store their data on local servers, using a hybrid approach (using both bare metal servers as well as a cloud services) can provide a more flexible option for storing data.

2. Hosting

When you're not sure where to host data, a cloud platform is a great way to minimize uncertainty. A hybrid cloud portfolio can support locally hosted options in either the UK or elsewhere in the EU, and cost-effective cloud options will help mitigate the risks associated with long-term investment or expensive migrations.

3. Security

Cloud technology has advanced greatly and now it is actually more secure and reliable than traditional on premise solutions. Many business owners who are a accustomed to using local servers hesitate to transition to the cloud for fear of security risks. They worry that having their information "out there" on the cloud will make it more susceptible to hackers.

4. Vulnerability to disasters

If you're only storing your data on local servers, you may be more susceptible to having your data affected by a natural disaster. Certain precautions may help alleviate this risks—such as backing up data, for example—but utilizing the cloud can provide even greater protection.

While the cloud is not without its risks—after all, the cloud is essentially a few servers united together on a software level—it does create another layer of protection in the event of a disaster.

5. Benefit for disaster recovery

Hosting systems and storing documents on the cloud provides a smart safeguard in case of an emergency. Man-made and natural disasters can damage equipment, shut off power and incapacitate critical IT functions. Supporting disaster recovery efforts is one of the important advantages of cloud computing for companies.

6. Increased long-term costs

Not moving to the cloud could cost your company money in the long run. While you do need to Pay for equipment with the cloud, costs are often more flexible because you can pay as you go depending on how much storage space you need,‘ On Demand’. Using this hybrid approach of combining cloud services and local dedicated servers,you can ensure you’re not paying for more Storage than you need.

7. Boosts cost efficiency

It reduces or eliminates the need for business to purchase equipment and build out and operate data enters. This presents a significant savings on hardware, facilities, utilities and other expenses required from traditional computing. Also, reducing the need for on-site servers, software and staff can trim the IT budget further.

8. Flexibility

Business have historically been tethered to wherever their equipment is located, because that's where they need to access all their information. This becomes a problem, though, when employees need to work outside the office because it may limit or eliminate their ability to work from home, meet with clients out in the field, or network away from their workspace.

With the cloud, however, users can bring their data with them wherever go. The cloud not only makes business more flexible, but it allows them to use their personal devices to access this information if need be.

9. Reduced agility

This ability to scale up or down can be critical for a business to stay agile and competitive. While Local servers may fit your needs now, what if you need to scale up as demand increases? By adding cloud services, you can add storage as you need it and pay as you go. This type of hybrid approach can adapt to your business's needs quickly, making it easier to meet demand as your company grows.

10. Limited technical support

Outside the cloud, your organization is limited to whoever is working inside your office. In the case of an emergency, you either have to hop your local professional can get the job done or hire a third-party company to help, which could be costly.

This risk is reduced in the cloud because you'll have the built-in support of experienced professionals, and you won't have to rely on anyone with minimal experience.

4. CONCLUSION

Encryption is the process through which data is protected from unwanted eyes. Encryption is the most effective form of data security, but unfortunately it is also an area that very few people know how to approach.

Encryption is the most effective way to achieve data security. To read an encrypted file, you must have access to a secret key or password that enables you to decrypt it, data

encryption among Corporates. The inability to protect data can destroy a company's reputation and no one wants to work with an organization that may allow sensitive information to fall into the wrong hands.

References

https://www.orchestrate.com

https://www.kwixand.com

NANOFLUIDS AND APPLICATIONS

1.Aryalakshmi C D
M.Sc. Scholar, Department of Mathematics, St. Thomas' College (Autonomous), Thrissur, Kerala.
2. Julie Andrews
Assistant professor, Department of Mathematics, St. Thomas' College (Autonomous), Thrissur, Kerala.

ABSTRACT

The dispersion of nanoparticles (1-100nm in size) in fluids result in a stable suspension called nanofluids. Metals, oxides and carbides are used as the nanoparticles in nanofluids. Nanofluids possess several unique properties such as thermal conductivity, thermal diffusivity, viscosity et cetera. Due to these properties, nanofluids are used in many fields to increase the output of many systems. This paper discusses about some of the applications of nanofluids in various fields.

Keywords: Nanofluids, heat transfer, tribological applications, biomedical applications, applications in solar energy.

1. INTRODUCTION

Today, our Science and Technology has reached in its highest level. Many advancements and new inventions are made day by day. In this scientific era, scientists are in search of inventions which are smaller in dimension and highly efficient in performance to save space and time. When the dimensions of these equipment's become smaller, the heat flux associated with it tends to large. So there arise the necessity for improving our cooling technology. But the cooling fluids available are poor conductors of heat and so we need to design fluids, which are good conductors of heat. In order to increase conductivity of fluids, powdered particles are added in it. But this leads to the settling of particles and to prevent settling, additional efforts are taken to stir the mixture continuously. The search for more efficient fluid led to the invention of nanofluids. Nanofluids are formed by suspending nanoparticles, which are less than 100nm in size in traditional fluids, such as water, oil, ethylene glycol etc. Unique thermal, optical, electrical and magnetic properties of nanoparticles add to the efficiency of nanofluids. So, a very small amount of these particles can alter the thermal conductivity of host fluids

Nanofluids (nanoparticle fluid suspension) is the term coined by Stephen U.S. Choi (1995). Nanofluids are designed to achieve the highest possible thermal properties at the smallest possible concentrations by the uniform dispersion and stability of the suspended nanoparticles in host fluids. The stable suspensions of nanoparticles in fluids help to increase heat transfer and enable cooling of systems. Stable and high conductive nanofluids are prepared using one step and two step production method.

In two step method, first we produce nanoparticles and these particles are dispersed in one of the base fluids. In single step method, the nanoparticles production and dispersion in base fluids occur simultaneously.

Nanoparticles used in nanofluids are of different materials, such as oxide ceramics (Al2O3, CuO), nitric ceramics (AlN, SiN), carbide ceramics (SiC, TiC), metals (Cu, Au, Ag), semiconductors (TiO2, SiC) and so on. These nanoparticles are produced by inert-gas condensation (IGC), mechanical milling, chemical precipitation, thermal spray and spray pyrolysis. Host liquids are water, ethylene glycol and oil.

2. APPLICATIONS

Many industries are facing thermal challenges, so there arise a need for ultrahigh-performance cooling. Conventional fluids are poor heat transferring fluids. Although particles are added to increase conductivity but they cause problems of increase in pressure drops, clogging and they settle down in the fluids. If we allow those particles to circulate in order to prevent settling, may cause damage to the walls of the pipes.

Nanofluids are designed as a heat transfer fluid with uniform distribution of nanometer sized particles in liquids to form stable suspensions. These are very dilute suspension of small quantity of nanoparticles in liquids. The larger surface area of these particles increases the heat transfer. Nanofluids can be used to improve heat transfer and energy efficiency in thermal control systems. The most important application of nanofluids is the application in thermal management of industrial and consumer product. Effective cooling is the main application of nanofluids that is as a coolant, nanofluids play its key role in the world of science and technology.

2.1. TRIBOLOGICAL APPLICATIONS

We can use nanofluids to develop better oils and lubricants. The tribological properties of lubricants, such as load carrying capacity, anti-wear and friction reducing properties between the moving mechanical components, are enhanced by the use of nanoparticles in lubricants. In this application, it is observed that surface modified nanoparticles stably dispersed in mineral oils are very effective in reducing wear and tear and increasing the load carrying capacity. Experiments were conducted on lubricant nanofluids containing IrO_2 and ZrO_2 nanoparticles. These experiments lead to conclude that these nanoparticles reduced friction noticeably on the surface of 100C6 steel.

2.2. BIOMEDICAL APPLICATIONS

Nanofluids can be used for variety of purposes other than cooling. Nanofluids are now developed for medical applications, especially in cancer therapy. Traditional methods for cancer treatment such as chemotherapy have certain side effects. Iron based nanoparticles can be used as scavengers for drugs or radiation without damaging the nearby healthy tissue by guiding the particles up the bloodstream to a tumor with magnets. Magnetic nanoparticles can manipulate the nanofluids using magnetic force. Magnetic nanoparticles are more adhesive.

to tumor cells so they absorb much more power than the micro particles in alternating current magnetic fields tolerable in human.

In a Nano drug supply system, the principal concern were the conditions for delivering uniform concentrations at the microchannel exit of the supplied nanodrugs. Depending on the levels of nanofluids, heat flux originates and the change in fluid velocity leads ascertain that drug delivery to living cells occurs at an optimal temperature, that is 37°C. The wall heat flux also produces a positive influence on drug concentration uniformity. This nanodrug concentration uniformity is affected by channel length, particle diameter and Reynolds number of both the nanofluid supply and main microchannel. The transport mechanisms depend on diffusion, longer channels, smaller particle diameters as well as lower Reynolds number yields a smooth drug delivery. The nanoparticles are excellent drug delivery vehicles because they are too small that the living cells absorb them when they arrive at the cell’s surface. Nanofluids also help in surgery, by cooling the surgical portion and thereby helping the patient reducing the risk of organ damage and saving the patient's life or increasing the chance of survival.

Nanofluids are involved in many biomedical applications such as magnetic cell separation, drug delivery, hyperthermia and contrast enhancement in magnetic resonance imaging. Surface coating of nanoparticles and the colloidal stability of biocompatible water-based magnetic nanofluid are the two factors that affect the successful application of nanofluids.

In contrast with cooling, nanofluids can be used to produce higher temperature around the tumor, to kill the cancerous cells without affecting nearby healthy cells.

2.3. APPLICATIONS IN SOLAR ENERGY

Solar energy system is an eco-friendly form of energy. There the solar energy from the rays of sun is directly converted to electrical energy. Nanofluids, as a potential heat transfer fluid can effectively increase the thermal performance of solar devices. Nanoparticles acts as the absorption medium allowing nanofluid to directly absorb solar energy. In collectors and solar water heaters, the thermal performance of nanofluids is used. Nanofluids enhances the development of solar energy systems.

Other areas of applications of nanofluid technology include cooling small computers, and electronic devices which are used in military systems, airplanes, spacecrafts. As a fluid superconductor, nanofluids could be used as a working fluid to extract energy from earth core and produce large scale energies in power plant systems. It is shown that the combustion of diesel fuel mixed with aqueous aluminum nanofluid increased the total combustion heat while decreasing the nitric oxide and smoke in the exhaust emission from the diesel engine.

3. CONCLUSION

Nanofluids are important because they have various application in heat transfer and biomedicine and so on. As a coolant, nanofluid helps in the ultra-high performance of equipment’s. Colloidal suspensions which are also the nanofluids helps in biomedicine for various purposes. There is a chance of increase in the use of nanofluids in biomedical engineering and the biosciences. This paper is a review on the applications of nanofluids in biomedicine, tribological applications, applications in solar energy. The growth of nanofluids in the last two decades is the evidence that in the progression of science and technology, these colloidal suspensions have an inevitable service.

4. REFERENCES

[1] Jang S P & Choi S U S, 2004. Role of Brownian motion in the enhanced thermal conductivity of nanofluids,Appl.Phys.lett. ,84 4316-4318

[2] Xuany & Li Q ,2003 Investigation on convective heat transfer and flow features of nanofluids,J. Heat Transf.,125, 151-155

[3] T.P. Otanicar, J. Golden, 2009, Comparative environmental and economic analysis of conventional and nanofluid solar hot water technologies, Environ. Sci. Technol. 43, 6082-6087

[4] OmidMahian, Ali Kianifar, Soteris A, Kalogirou, loan Pop , Somchai Wongwises,2013, a review of the applications of nanofluids in solar energy , international journal of Heat and Mass Transfer, 57

[5] Jordan, A., R. Scholz, P. Wust, H. Fähling, and R. Felix (1999). Magnetic fluid hyperthermia (MFH): cancer treatment with ac magnetic field induced excitation of biocompatible superparamagnetic particles, J. Magn. Magn. Mater., 201(1-3): 413-419.

[6] Que, Q., J. Zhang, and Z. Zhang (1997). Synthesis, structure and lubricating properties of dialkyldithiophosphate-modified Mo-S compound nanoclusters, Wear, 209(1-2): 8-12

[7] Delgratta C, Dellapenna S, Battista P, Didonato L, Vitullo P, Romani G and Diluzio S, 1995, Detection and counting of specific cell populations by means of magnetic markers linked to monoclonal antibodies Phys. Med. Biol. 40 671-81

[8] Edelstein R L, Tamanaha C R, Sheehan P E, Miller M M, Baselt D R, Whitman L J and Colton R J, 2000, The BARC biosensor applied to the detection of biological warfare agents Biosensors Bioelectron. 14 805-13

[9] Hofmann W K, de Vos S, Komor M, Hoelzer D, Wachsman W and Koeffler H P, 2002, Characterization of gene expression of CD34+ cells from normal and myelodysplastic bone marrow Blood 100 3553-60

[10] Sarit K Das, Stephen U S Choi, Wenhua Yu, T. Pradeep, Nanofluids Science and Technology, A John Weily & Sons, Inc, Publication 2007

APPLICATIONS OF MATHEMATICS

ASHIK JOSEPH

B.Sc. Scholar, Department of Statistics, St. Thomas college(autonomous)Thrissur, Kerala India

Email: ashikjoseph697@gmail.com.

MATHEMATICS IN SCHOOL AND WORK

Students can't avoid math. Most take it every day. However even in history and English classes they may need to know a little math. Whether looking at time expanses of decades, centuries or eras or calculating how they'll bring that Bin English to an A, they'll need some basic math skills. Jobs in business and finance may require sophisticated knowledge of how to read profit and earning statements or how to decipher graph analyses.

However, even hourly earners will need to know if their working hours multiplied by their rate of pay accurately reflects their paychecks.

MATHEMATICS IN FINANCE

Many mathematical a statistical application are yet to be improved to make into account of the intrinsic complexities in finance and related fields. Many of the statistical tests it seems do not sufficiently discriminate.

For example, statistical tests usually fail to contradict the random walk hypothesis for prices. It is certain that more work is needed to cope with the large effects of noise in financial time series analysis .A number other aspects it seems needs work such as the assumption that participants act rationally and aim to maximize returns The works on neural psychological and behavioral finance may help provide significant insights and advances in thinking .It is certain that if all of the above are incorporated into the modelling process a higher level of mathematics will be required to deal with aspects such as “real” market participants ideas of random walks ,market interdependencies correlations, and so on Tularam.G.A (2013).Math-mastics in finance e-journal of Business Education and Scholarship of teaching

MATHEMATICS IN MATERIALS SCIENCE

Materials sciences is concerned the synthesis and manufacture of new materials, the modification of materials ,the understanding and prediction of material properties ,and the evolution and control ot these properties over time period .Until recently materials science was primarily an empirical study in metallurgy ,ceramics, and plastics .Today it is a vast growing body of knowledge based on physical sciences ,engineering and mathematics .For example, mathematical models are emerging quite reliable in the synthesis and manufacture of polymers .Some of these models are based on a diffusion equation in finite or infinite dimensional spaces

Simpler but more phenomenological models of polymers are based on continuum Mechanics with added terms to account for ‘memory’. Stability and singularity of solutions are important issues for materials scientists. The mathematicians are still lacking even for these simpler models. Another example, is the study of the formation of cracks in materials. When a uniform elastic body is subjected to high pressure, cracks initiate, how they evolve, and questions that are still being researched.

MATHEMATICS IN BIOLOGY

Mathematical models are also emerging in the biological and medical sciences. For example, in physiology, consider the kidney. One million tiny tubes around the kidney called nephrons, have the task of absorbing salt from the blood into the kidney. They do it through contact with blood vessels by a transport process in which osmotic pressure and filtration by a role. Biologists have identified the body tissues and substances, which a are involved in this process, but the precise rules of the process are only barely understood. A simple mathematical model of the renal process shed some light on the formation of urine and on decisions made by the kidney whether. For example, to execrate a large of diluted urine or a small volume of conc. urine. A more complete model may include PDE, stochastic equations, fluid dynamics elasticity theory, filtering theory and control theory, and perhaps other tools.

Other topics in physiology where recent mathematical studies have already made some progress include heart dynamics, calcium dynamics the auditory process, cell adhesion and motility and biofluids .Other areas where mathematics is poised to make important progress include the growth process in general and embryology in particular cell signaling, immunology ,emerging and remerging infectious diseases and ecological issues such as global phenomena in vegetation, modelling animal grouping and the human brain.

MATHEMATICS IN ARMY

Recent trends in army in mathematics research in the USA ARMY have been influenced by lessons learnt during Comat in Bosnia. The USA Army could not bring heavy tanks in time and helicopters were not used to avoid casualty. Also, there is need for lighter systems with same or improved requirement as before. Breakthroughs are urgently needed and mathematics research is being funded with a hope to get the urgently needed systems' future automated systems are complex and nonlinear, they will likely be multiple units, small in size, light in weight, very sufficient, in energy utilization and extremely fast in speed and will likely be self-organized and self-coordinated to perform special tasks.

During the last 50 years, developments in mathematics, in computing and communication technologies have made it possible for most of the breath-taking discoveries in basic sciences, for the tremendous innovations and inventions in engineering sciences and technology and for the great achievements and breakthroughs in economics and life sciences. These have led to the emergency of many new areas of mathematics and enabled areas that were dormant to explode. Now every branch of mathematics has a potential for applicability in other fields of mathematics and their disciplines. All these, have pose a big challenge on the on the mathematics curricula at all levels of the education systems, teacher preparation and pedagogy. The 21st century mathematics thinking is to further strengthen efforts to bridge the division lines within mathematics, to open more for other disciplines and to foster the line of inter-discipline research.

MATHEMATICS IN BANKING

A lot of teens do not have bank accounts, but you still do banking. You need to know how to manage your allowances so that you can afford the best that you can get on what you have. Otherwise, you may find yourself without money for the essentials like stationary after buying the luxuries like ice cream. Mathematics in gardening even doing something as mundane as gardening requires a basic math skill. If you need to plant or sow new seeds or seedlings you need to make row or count them out or even make holes. So even without thinking you are doing math. Measuring skills is always needed, and calculations are important when doing something new in the garden.

MATHEMATICS IN KITCHEN

Whatever you do in the kitchen requires math, like counting the no. of teaspoons of sugar thar are just right for you in your tea or coffee or complicated cooking and baking. Even just using the stove, microwave kettle is basic math skills in action.

MATHEMATICS IN AGRICUTURE

Fertilizers are needed to harvest products in optimum quantity. In order to provide optimum levels to the plant of concern, you should know levels of elements available in your soil.

Nitrogen and phosphorous are 2 fertilizers that should be supplied by fertilizers. NH_4 $+NO_3$ increased wheat yields 7to 47% in 14 studies. Animal manures and other types of organic waste may be important sources of nitrogen for optimum plant growth. The amount of nitrogen supplied by the application of manure varies with the type of livestock, handling, rate applied, and method of application. Because the nitrogen form and content of manures varies notably, an analysis of manure is recommended to improve nitrogen management. Optimum levels should know. Another application is irrigation water quality such as sodium adsorption ratio to compute your irrigation water quality. There are formulas developed to compute SAR and adj SAR.

Reference

Application of Mathematics in Different Fields. (2020, January 15). GradesFixer. Retrieved March 6, 2021, from https://gradesfixer.com/free-essay-examples/application-of-mathematics-in-different-fields/

TWO IMPORTANT APPLICATIONS OF MATHEMATICS IN INVESTMENT DECISION MAKING

LIJO JOSE
Assistant Professor, Department of Commerce (self-finance), St. Thomas College (Autonomous), Thrissur.

ABSTRACT

Making a good investment decision is very important for an investor. But often people do not know the proper way to do it. In light of this, the purpose of this paper is to introduce two mathematical tools that will help any individual to make a good investment decision. Knowledge of this application in mathematics is very helpful to analyse each investment opportunity.

Keywords

Applied Mathematics, Time Value of Money, Present value of Money, Future value of Money

What is Applied Mathematics?

Applied mathematics involves the application of mathematics to problems which arise in various areas, e.g., science, engineering or other diverse areas, and/or the development of new or improved methods to meet the challenges of new problems.

We view applied math as the application of mathematics to real-world problems with the dual goal of explaining observed phenomena and predicting new, as yet unobserved, phenomena. Therefore, the emphasis is on both the mathematics, e.g., the development of new methods to meet the challenges of new problems, and the real world.

What is the Time Value of Money?

The time value of money is a basic financial concept that holds that money in the present is worth more than the same sum of money to be received in the future. This is true because money that you have right now can be invested and earn a return, thus creating a larger amount of money in the future. (Also, with future money, there is the additional risk that the money may never actually be received, for one reason or another.) The time value of money is sometimes referred to as the net present value (NPV) of money.

Future Value of Money

FV is one of the most important concepts in finance, and it is based on the time value of money. Investors need to know what the FV of their investment will be after a certain period of time, calculated based on an assumed growth rate.

For instance, a $1,000 investment that pays a fixed interest rate of 5% will be $2,654 after 20 years, all things being equal. Therefore, the FV uses a single upfront investment and a constant rate of growth during the time horizon of the investment. On the downside, the FV is not adjusted for high inflation or changes in the interest rates, which are factors with a negative impact on any investment.

A specific formula can be used for calculating the future value of money so that it can be compared to the present value:

FV = $PV \times (1 + i)^n$

where:

PV = the present value of the investment or the beginning value

FV = the future value of the investment after or the number of periods the deposit is invested

i = the interest earned on the investment

n = the number of time periods in months the deposit remains invested

Example:

If you deposit ₹100 at the beginning of a year with the interest rate of 5% and if the number of years is 1, then you can read the formula as follows:

"The future value (FV) at the end of one year equals the present value (₹100) plus the value of the interest at the specified interest rate i.e., ₹5 (5% of ₹100)."

i.e., FV = 100 + 5 = 105

What a Future Value Formula Works

To determine the value of your investment at the end of 5 years, you would change your calculation to include an exponent representing the 5 periods:

Future value of ₹100 after 5 years at rate of 5% interest rate is

At end of 1^{st} year = 100 + 5 = 105

At end of 2^{nd} year = 105 +(105 × 5%) = 110.25

At end of 3^{rd} year = 110.25 + (110.25 × 5%) = 115.7625

At end of 4^{th} year = 115.7625 + (115.7625 × 5%) = 121.5506

At end of 5^{th} year = 121.5506+ (121.5506 × 5%) = 127.6281

OR

Application of FV $= PV \times (1+i)^n$

$= 100 \times (1+.05)^5$

$= 100 \times 1.2763$

$= 127.62$

What Is Present Value (PV)?

Present value (PV) is the current value of a future sum of money or stream of cash flows given a specified rate of return. Future cash flows are discounted at the discount rate, and the higher the discount rate, the lower the present value of the future cash flows. Determining the appropriate discount rate is the key to properly valuing future cash flows, whether they be earnings or debt obligations.

KEY TAKEAWAYS

- Present value states that an amount of money today is worth more than the same amount in the future.
- In other words, present value shows that money received in the future is not worth as much as an equal amount received today.
- Unspent money today could lose value in the future by an implied annual rate due to inflation or the rate of return if the money was invested.
- Calculating present value involves assuming that a rate of return could be earned on the funds over the period.

$PV = FV / (1+r)^n$

- PV is Present Value
- FV is Future Value
- **r** is the interest rate (as a decimal, so 0.10, not 10%)
- **n** is the number of years

Example:

To get ₹ 5000 p.a for next 4 years what should invest now if the market rate is 5%?

$$= \quad \frac{5000}{(1+.05)^1} + \frac{5000}{(1+.05)^1} + \frac{5000}{(1+.05)^1} + \frac{5000}{(1+.05)^1}$$

= 4762 + 4545 + 4310 + 4132

= ₹ 17,749

CONCLUSION

Thus, it is evident that mathematical methods and tools become crucial part of the Investment Decision Making. Mathematical formulae help investors to forecast their return and to choose good investment opportunities. It also helps to avoid investment in overpriced securities and protect the selling of securities from underpriced. In the light of above, I conclude that knowledge of mathematics is very helpful to take good investment decisions.

References

[1] Bhattacharya, Harshika's: Working Capital Management: Strategies and Techniques; Prentice Hall of India, New Delhi.

[2] Chandra, Prasanna: Financial Management; Tata McGraw Hill, Delhi

[3] Pandey, I.M.: Financial Management, Prentice Hall of India, New Delhi.

[4] Khan M.Y. and Jain P.k.: Financial Management; Tata McGraw Hill, Delhi.

[5] Vanhorne, J.C.: Financial Management and Policy; Prentice Hall of India, New Delhi.

[6] Ravi M Kishore: Fundamentals of Financial Management (Tax man publication)

MATHEMATICAL MODELING OF SUPERCONDUCTING TRANSMISSION LINES USING SPECTRAL DOMAIN METHODOLOGY

Jolly Andrews, Department of Physics, Christ College Irinjalakuda, Thrissur-680125, Kerala, India Email: jmalieckkal123@gmail.com
V P Joseph, Department of Physics, Christ College Irinjalakuda, Thrissur-680125, Kerala, India

ABSTRACT

We present a modeling methodology to study the electromagnetic wave propagation in superconducting inhomogeneous transmission lines using Spectral Domain Method (SDM). The inhomogeneous transmission lines like microstrip line, slot line and coplanar waveguide (CPW) are mathematically modeled by matching the electromagnetic boundary conditions. The dyadic Green's functions derived for each of these transmission lines are modified to incorporate the impact of superconducting strip in electromagnetic wave propagation. The unknown current or field components are expanded in terms of known basis functions as the case may be. Using Galerkin procedure we compute the dispersion relation. Depending upon the type of the theory used to model the High Temperature Superconducting (HTS) vortex movements, we can study the different aspects of the wave propagation. Modified two-fluid model proposed by Coffey and Clem (CC) is taken as a case study in CPW to study the propagation parameters.

1. INTRODUCTION

The spectral domain method (SDM), first introduced by Itoh and Mittra in 1973, is an elegant tool which uses Galerkin's procedure for the computation of the propagation constant [1]. The SDM is now used for the analysis of a wide range of structures like planar resonators, coupled microstrip lines, microstrip discontinuities etc. The SDM also finds application in the full wave analysis of periodic structures, finline structures, planar structures on semi-insulating and semi-conductor substrates, superconducting structures etc [1]. In this paper, we present a modeling technique used to study the propagation parameters of microstip line, slotline and coplanar waveguide (CPW). Normal metal strip is replaced with a High Temperature Superconducting (HTS) strip due to its signal transmission advantages. This makes the modeling mathematically challenging owing to the presence of vortices present in the superconducting strip. In the absence of a comprehensive microscopic theory of HTS materials, we use the phenomenological modified two-fluid theory developed by Coffey and Clem (CC) [2]. CC model studies the microwave response of the vortex effects in a linear response regime [2]. The modeling results of Coplanar waveguide (CPW) are presented and analyzed. We take the material parameters of YBCO, though our simulation can be performed using any such HTS material. We take the HTS strip as an isotropic material for the sake of

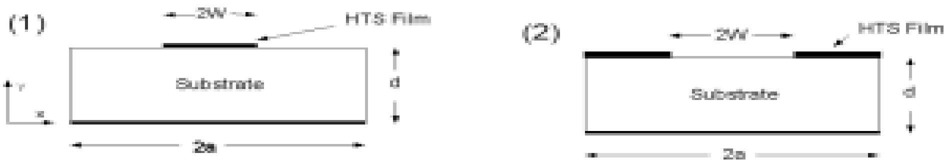


Fig. 1. The layout of microstrip line (1) and slotline (2) with superconducting strip

simplicity. The substrate taken is sapphire and its permittivity is taken as isotropic. The dispersion relation is obtained using Muller complex root finding method. The paper explains ¨ the modeling technique of inhomogeneous transmission lines using HTS material by incorporating vortex dynamical effects in a self-consistent manner.

2. SPECTRAL DOMAIN MODELING

Spectral Domain analysis is done in the Fourier space. By convention, we define the Fourier transform as

$$\tilde{F}(\alpha) = \int_{-\infty}^{+\infty} F(x) exp[\mathrm{i}(\alpha \mathrm{x})] \mathrm{dx} \qquad (1)$$

where α is the transform variable. In the presence of side walls, the integration limits get modified as ±a where a is the cross-sectional length of the transmission line. In Fig. 1 we sketch the geometry of a HTS microstrip transmission line and a HTS slotline. In Fig. 2 we have the cross section of a HTS Coplanar waveguide (CPW).

A. Microstrip Line

We first derive the Green's functions by treating the strip as a normal conductor. We define $\gamma^2 = \alpha^2 + \beta^2 - k^2 \; with \; k^2 = \omega^2 \mu \epsilon$ when the time and the z dependences are expressed as $e^{i(\omega t - \beta z)}$. Here μ is the permeability and is the permittivity of the medium under consideration. In conventional space domain analysis, the microstrip line is analyzed by formulating the following coupled homogeneous integral equation as

$$\int [Z_{zz} J_z(x') + Z_{zx} J_x(x')] dx' = E_z(x) \qquad (2a)$$

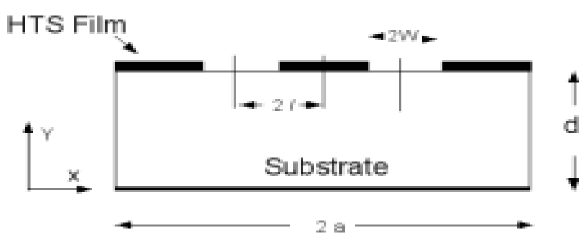


Fig. 2. The layout of coplanar waveguide with superconducting strip

$$\int [Z_{xz} J_z(x') + Z_{xx} J_x(x')] dx' = E_x(x) \qquad (2b)$$

where E_z and Ex are unknown electric fields, J_x and J_z are the current components on the strip and Green functions $Z_{zz}s$ etc are function of β, the propagation constant. We obtain the Fourier transforms of the coupled integral equations as

$$\tilde{Z}_{zz}(\alpha, d)\tilde{J}_z(\alpha) + \tilde{Z}_{zx}(\alpha, d)\tilde{J}_x(\alpha) = \tilde{E}_z(\alpha, d) \qquad (3a)$$
$$\tilde{Z}_{xz}(\alpha, d)\tilde{J}_z(\alpha) + \tilde{Z}_{xx}(\alpha, d)\tilde{J}_x(\alpha) = \tilde{E}_x(\alpha, d) \qquad (3b)$$

where $\sim$ represent the Fourier transformed quantities. A detailed description of the formulation of the $Z_{(zz)s}$ is given elsewhere [1]. With the help of the electromagnetic boundary conditions and by using mathematical manipulations whereby we eliminate the magnetic field components, we can derive a relationship between the Fourier transforms of the electric fields and those of the current distributions as a set of algebraic equations expressed as

$$\begin{pmatrix} \tilde{E}_z \\ \tilde{E}_x \end{pmatrix} = \begin{pmatrix} \tilde{Z}_{zz} & \tilde{Z}_{zx} \\ \tilde{Z}_{xz} & \tilde{Z}_{xx} \end{pmatrix} \begin{pmatrix} \tilde{J}_z \\ \tilde{J}_x \end{pmatrix} \qquad (4)$$

The impedance matrix elements $\tilde{Z}$s in the Eq. 4 are defined as follows:

$$\tilde{Z}_{zz} = -\frac{1}{\alpha^2 + \beta^2}\left[\beta^2 \tilde{Z}_e + \alpha^2 \tilde{Z}_h\right] \qquad (5a)$$
$$\tilde{Z}_{zx} = -\frac{\alpha\beta}{\alpha^2 + \beta^2}\left[\tilde{Z}_e - \tilde{Z}_h\right] \qquad (5b)$$
$$\tilde{Z}_{xz} = \tilde{Z}_{zx} \qquad (5c)$$
$$\tilde{Z}_{xx} = -\frac{1}{\alpha^2 + \beta^2}\left[\alpha^2 \tilde{Z}_e + \beta^2 \tilde{Z}_h\right] \qquad (5d)$$

where we have

$$\tilde{Z}_e = \frac{\gamma_{y1}\gamma_{y2}}{\gamma_{y1} C t_2 + \gamma_{y2} C t_1} \qquad (6a)$$
$$\tilde{Z}_h = \frac{1}{\gamma_{z1} C t_1 + \gamma_{z2} C t_2} \qquad (6b)$$

with the terms defined as $Ct_1 = coth[\gamma 1(h1 - d)]$ and $Ct_2 = coth[(\gamma 2)d]$. Here $y\hat{} = i\omega\epsilon$, $z\hat{}$ = iωμ, $\gamma_{yj} = \frac{\gamma j}{yj}$ and $\gamma_{zj} = \frac{\gamma j}{zi}$ for j = 1,2 and refers to the regions above the substrate and the substrate respectively. In the next step we modify the dyadic Green functions by incorporating the HTS contribution [3], [4], [5], [6], [7]. We first expand the unknown Electric field E˜ as

$$\begin{pmatrix} \tilde{E}_x \\ \tilde{E}_z \end{pmatrix} = \begin{pmatrix} \tilde{E}_x^e + Z_s\tilde{J}_x \\ \tilde{E}_z^e + Z_s\tilde{J}_z \end{pmatrix} \qquad (7)$$

where $E^{\tilde{}e}$ is the Electric field distribution at the interface excluding the region of the strip, Zs is the surface impedance due to HTS material and $J^{\tilde{}}_{x,z}$ represent the strip current. Then our Fourier transformed coupled algebraic equation derived as Eq. 4 with the impedance Green's function elements will be modified by incorporating the surface impedance Z_s of the HTS strip as [3]

$$\begin{pmatrix} \tilde{E}_z^e \\ \tilde{E}_x^e \end{pmatrix} = \begin{pmatrix} \tilde{G}_{zz} & \tilde{G}_{zx} \\ \tilde{G}_{xz} & \tilde{G}_{xx} \end{pmatrix} \begin{pmatrix} \tilde{J}_z \\ \tilde{J}_x \end{pmatrix} \qquad (8)$$

where we have

$$\begin{pmatrix} \tilde{G}_{zz} & \tilde{G}_{zx} \\ \tilde{G}_{xz} & \tilde{G}_{xx} \end{pmatrix} = \begin{pmatrix} \tilde{Z}_{zz} - Z_s & \tilde{Z}_{zx} \\ \tilde{Z}_{xz} & \tilde{Z}_{xx} - Z_s \end{pmatrix} \qquad (9)$$

The unknown strip current $\tilde{J}_z$ and $\tilde{J}_x$ are expanded in terms of known basis functions $\tilde{J}_{zm}$ and $\tilde{J}_{xm}$ as

$$\tilde{J}_z = \sum_{m=1}^{N} c_m \tilde{J}_{zm}(\alpha) \qquad (10a)$$

$$\tilde{J}_x = \sum_{m=1}^{M} d_m \tilde{J}_{xm}(\alpha) \qquad (10b)$$

In the next step we will apply the Galerkin's procedure. We substitute Eq. 10 in Eq. 8 and take the inner products of the resultant equations with the known basis functions. Now taking the scalar product with the same set of basis functions will lead to an equation of the form [G][C] = 0, where [C] is the column matrix of the expansion coefficients. The equation det [G] = 0 will give the dispersion relation. In computation M and N can be kept minimum by selecting suitable basis functions [1], [5].

B. Slotline.

Slotline structure is complimentary to the microstrip line (Fig. 1). We notice that the field and the boundary conditions are same as those of the microstrip line and hence the integral equations are the same as that are derived before for a microstrip line. We see that in the case of slotline, the strip width is larger than the slot width compared with that of the microstrip line and hence we use the Galerkin procedure to the unknown Electric field E of the slot [1], [4]. Accordingly, our matrix equation can be written as.

$$\begin{pmatrix} \tilde{Y}_{xx} & \tilde{Y}_{xz} \\ \tilde{Y}_{zx} & \tilde{Y}_{zz} \end{pmatrix} \begin{pmatrix} \tilde{E}_x \\ \tilde{E}_z \end{pmatrix} = \begin{pmatrix} \tilde{J}_x \\ \tilde{J}_z \end{pmatrix} \qquad (11)$$

where [Y]'s are the admittance elements. In the next step we modify the dyadic Green's functions by incorporating the HTS contribution by using Eq. (7). We thus have

$$\begin{pmatrix} \tilde{Y}'_{xx} & \tilde{Y}'_{xz} \\ \tilde{Y}'_{zx} & \tilde{Y}'_{zz} \end{pmatrix} \begin{pmatrix} \tilde{E}^e_x \\ \tilde{E}^e_z \end{pmatrix} = \begin{pmatrix} \tilde{J}_x \\ \tilde{J}_z \end{pmatrix} \qquad (12)$$

The electric field basis functions are in the same form as that of the current basis functions and by using Galerkin's procedure we obtain the propagation parameter

C. Coplanar Waveguide.

The cross section of Coplanar Waveguide (CPW) is given in Fig. 2. Since there are two slots for the CPW, the electric field distributions over them are not symmetrical with respect to the center of each slot. The resulting basis functions for the fourier transform domain may be written as with lengths w and l as given in Fig. 2 as $E^{\sim}_{xn}(\alpha)$ and $E^{\sim}_{zn}(\alpha)$. Accordingly, the three basis functions formed are $E^{\sim}_{x1}$ for n = 0, $E^{\sim}_{x2}$ for n = 1 and $E^{\sim}_{z1}$ for n = 1 whose detailed derivation is given elsewhere [1], [7]. Applying the Galerkin's procedure using the three basis functions $E^{\sim e}_{x1}$, $E^{\sim e}_{x2}$, $E^{\sim e}_{z1}$ we will get a 3 × 3 matrix who's determinant when equated to zero will give us the propagation constant [1], [7].

3. COFFEY-CLEM MODEL.

We use the modified two-fluid theory introduced by Coffey and Clem (CC) to incorporate the HTS contribution of the transmission lines [2], [8], [9]. The remarkable feature of CC model is that it incorporates the effects of the fluxon motion, the pinning, flux flow and the flux creep in a self-consistent manner. The self-consistently determined penetration depth λ˜ = λ˜(ω, B, T) which accounts for the fluxon motion is given in terms of the normal fluid skin depth δ_{nf}, the complex effective skin depth $\tilde{}\delta_{vc}$ and the static-field and temperature dependent penetration depth λ(B, T) as [2].

$$\tilde{\lambda}(\omega, B, T) = \left(\frac{\lambda^2(B,T) - (\mathrm{i}/2)\tilde{\delta}^2_{\mathrm{vc}}}{1 + 2\mathrm{i}\lambda^2\delta^{-2}_{\mathrm{nf}}}\right)^{1/2}. \qquad (13)$$

λ(B, T) determines the superfluid response while δ_{nf} determines the strength of the normal fluid response [2]. $\tilde{}\delta_{vc}$ takes into account the motion of the vortices and is defined as $\tilde{}\delta^2_{vc}(\omega, B, T) = 2\tilde{}\rho v/\mu 0\omega$ where the effective resistivity $\rho\tilde{}_v\,(\omega, B, T) = B\varphi_0\mu\tilde{}_v(\omega, B, T)$ and $\mu\tilde{}_v\,(\omega, B, T)$ is the complex dynamic vortex mobility [2].

The complex surface impedance of the superconductor Z_s is given by $Z_s = \sqrt{i\omega\mu/\sigma\tilde{}}$ and in the case of the thin strip cases [3] we use $Zs = [h(\sigma n - i\sigma_{sc})]^{-1}$, where h is the superconducting strip. The rf complex resistivity appearing in the expression $E = \tilde{}\rho J$, is related to $\lambda\tilde{}\ via\ \rho\tilde{} = i\mu_0\omega\,\tilde{}\lambda 2$ with the complex conductivity $\sigma\tilde{} = 1/\rho$.

4. NUMERICAL RESULTS AND DISCUSSION

Numerical computation is performed on the basis of the above equations. As an example, we take the results for CPW. The material parameters used are the typical values of high T_c superconducting system of YBCO. $T_c = 92\,K, \lambda_0 = 140\,nm, \rho_n(T) = 1.1 \times 10^{-8}T + 2 \times 10^{-6}\Omega\,m, U = 0.15\,eV$.

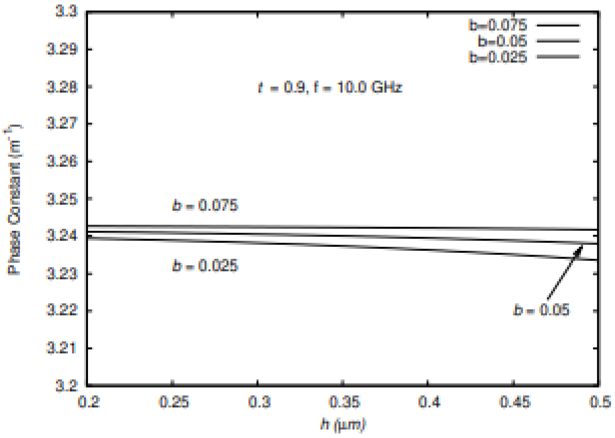


Fig. 3. The variation of the phase constant with the superconducting strip thickness for $T = 82.8$ K ($t = 0.9$) in the fields of 2.8, 5.6 and 8.4 T ($b = B/B_{c2}(0) = 0.025, 0.05, 0.075$) for the frequency 10.0 GHz

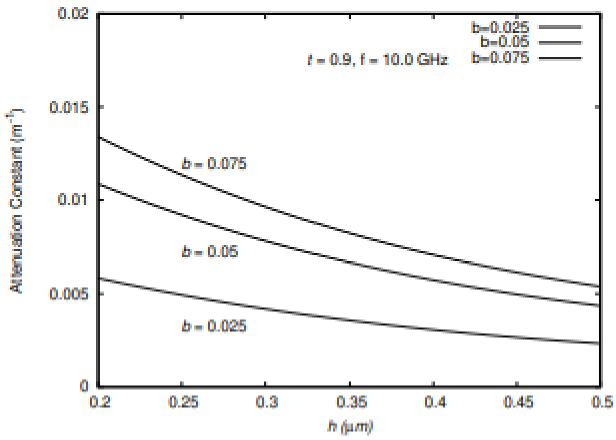


Fig. 4. The variation of the attenuation constant with the superconducting strip thickness for $T = 82.8$ K ($t = 0.9$) in the fields of 2.8, 5.6 and 8.4 T ($b = B/B_{c2}(0) = 0.025, 0.05, 0.075$) for the frequency 10.0 GHz

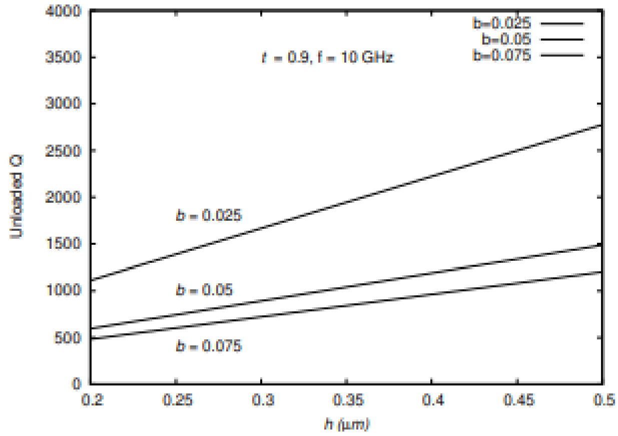


Fig. 5. The variation of the unloaded Q value with the superconducting strip thickness for T = 82.8 K (t = 0.9) in the fields of 2.8, 5.6 and 8.4 T ($b = B/B_{c2}(0) = 0.025, 0.05, 0.075$) for the frequency 10.0 GHz

$\kappa_{p_0} = 2.1 \times 104\ N/m$ and $B_{c2} = 112\ T$. The substrate thickness d is taken as 10 μm with permittivity 9.8, typical of sapphire. The width of the slot w is taken as 8 μm whereas the distance l is taken as 10 μm. The width of the substrate a is 40 μm. The applied magnetic field B is varied from 2.8 to 8.4 T and the frequency used in the study is 10.0 GHz. For applying the strip line equation of Pond [3] we take h < λ(B, T). The Coffeyclem model imposes two further conditions [2]; the inter vortex distance $a0$ should be lower than the strip thickness h and the penetration depth λ(B, T). All these constraints are fairly well satisfied in the numerical simulation.

In Fig. 3 we have plotted the variation of phase constant with the superconducting strip thickness which is varied from 0.2 μm to 0.5 μm for the three applied field values of 2.8, 5.6 and 8.4 T. The study is done at a reduced temperature t = 0.9 and the frequency used is 10 G Hz. We observe from the numerical value of the phase constant that the variation is quite negligible as the strip thickness is varied. Incidentally, at such low strip thickness we find a slight lowering in phase value as we enhance the strip thickness and this variation is lower for a higher field value. For a fixed field value we find a higher phase value for a higher applied field. The reason is due to the enhanced number of vortices and their movements at high temperature. But we observe that dispersion is largely negligible for the three different fields and for the strip variations.

In Fig. 4 we present the variation of the attenuation constant with the strip thickness. We see that as the strip thickness is gradually enhanced, the value of attenuation is gradually decreasing. We observe that for a fixed strip thickness we find a lower value of attenuation for a lower field value. As we increase the magnetic field, more vortices are induced into the HTS material and this result in the enhancement of the value of attenuation. The results are in tune with that of the microstrip case where we find a significant lowering in the attenuation value [5]. But the phase value has remained largely constant compared with that of the microstrip line. In Fig. 5 we present the Unloaded Q value with the varying strip thickness h. The Q value is calculated from the corresponding phase propagation constant and attenuation factor [5]. As is expected we find a higher Q value for a higher strip thickness. For a fixed strip thickness we find a lower value of Q for a higher field value. The result shows that the Unloaded Q value largely depends on the variation of the attenuation constant and has very little influence of the phase constant variation. The lowering of attenuation has resulted in enhancement of the Q value. The impact of strip thickness on Q value is more at low field value and this shows the influence of vortex effects on signal transmission. The experimental results of Song and his group show similar results [10].

5. CONCLUSION

A modeling methodology has been presented in this paper for the inhomogeneous microwave transmission lines like microstrip, slotline and CPW. The dyadic Green's functions are derived by incorporating the surface impedance derived from the HTS material using CC model. SDM in Galerkin's procedure is employed to obtain the propagation parameters.

The methodology employed in this paper can be used to model transmission lines with multiple substrates and with different geometries. The study also shows the relationship between vortex motion and microwave transmission.

REFERENCES

[1] T. Uwano and T. Itoh, Numerical Techniques for Microwave and Millimeter WavePassive Structures, T. Itoh, Ed. John Wiley, New York, 1989.

[2] M. W. Coffey and J. R. Clem, "Unified theory of effects of vortex pinning and flux creep upon the rf surface impedance of type-ii superconductors," Phys. Rev. Lett., vol. 67, pp. 386–389, Jul 1991.

[3] J. Pond, C. Krowne, and W. Carter, "On the application of complex resistive boundary conditions to model transmission lines consisting of very thin superconductors," Microwave Theory and Techniques, IEEE Transactions on, vol. 37, no. 1, pp. 181 –190, jan 1989

[4] S. Mathew, A. R, and V. Mathew, "Propagation characteristics of superconducting slot line based on ferroelectric substrate," Journal of Superconductivity and Novel Magnetism, vol. 26, no. 9, pp. 2899–2904, 2013.

[5] J. Andrews and V. Mathew, "Numerical study of vortex effects in superconducting microstrip lines," Physica C: Superconductivity, vol. 471, no. 1112, pp. 338 – 343, 2011.

[6] J. Andrews and V. Mathew, "Magnetic field induced properties of typeii superconducting microstrip resonators," Superconductor Science and Technology, vol. 25, no. 2, p. 025004, 2012

[7] J. Andrews and V. Mathew, "Vortex dynamics effects on microwave propagation in high temperature superconducting coplanar waveguides," Journal of Applied Physics, vol. 114, p. 163914, 2013.

[8] H. R. Mohebbi and A. H. Majedi, "Cad model for circuit parameters of superconducting-based hybrid planar transmission lines," Superconductor Science and Technology, vol. 22, no. 12, p. 125028, 2009.

[9] M. Golosovsky, M. Tsindlekht, and D. Davidov, "High-frequency vortex dynamics in yba2 cu3o7," Superconductor Science and Technology, vol. 9, no. 1, p. 1, 1996.

[10] C. Song, M. P. DeFeo, K. Yu, and B. L. T. Plourde, "Reducing microwave loss in superconducting resonators due to trapped vortices," Applied Physics Letters, vol. 95, no. 23, pp. 232 501 –232 501–3, dec 2009.

Publications Division, St. Thomas College Thrissur

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- Hentry o Pollack

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PG & Research Department of Mathematics
St. Thomas College (Autonomous), Thrissur

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