

Executive Summary of Minor Research Project on

COMPARISON OF EFFICIENCY OF GOVERNMENT SCHOOLS TO PRIVATE INSTITUTIONS

Dr. T. B. Ramkumar

Dept of Statistics, St.Thomas College, Thrissur, Kerala.

E-mail: rktmidhuna@gmail.com

Data Envelopment Analysis (DEA) is a methodology based on an interesting application of linear programming. DEA involves an alternative principle for extracting information about a population of observations, so called decision making units (DMUs) that are described by the same quantitative characteristics. The problem is complicated by the fact that the DMUs consume a variety of identical inputs and produce a variety of identical outputs. The performances of DMUs are assessed in DEA using the concept of efficiency or productivity, which is the ratio of total outputs to total inputs.

Efficiency = output /input

The best performing DMU is assigned an efficiency score of unity or 100 percent, and the performance of other DMUs vary between 0 and 100 percent relative to this best performance.

The main advantage of DEA is that it can readily incorporate multiple inputs and outputs to calculate technical efficiency. By identifying the 'peers' for organizations that are not observed to be efficient, it provides a set of potential role models that an organization can look in to, at first instance, for ways of improving its operations.

Limitation of DEA is sensitive to measurement error and it is not meaningful to compare the scores between two different studies

DEA Models

The efficiency measure indicates whether a firm uses the minimum quantity of inputs to produce a given quantity of outputs or maximizes the output quantity given a certain quantity of inputs. If x_{ij} , $i \in H$, denotes the quantity of input i used by DMU_j and y_{rj} , $r \in K$ the quantity of output r obtained, the efficiency of DMU_j is defined as

$$\theta_j = (u_1 y_{1j} + u_2 y_{2j} + \dots + u_m y_{mj}) / (v_1 x_{1j} + v_2 x_{2j} + \dots + v_s x_{sj})$$

$= \sum_{r \in K} u_r y_{rk} / \sum_{i \in H} v_i x_{ij}$ where u_1, \dots, u_m associated with the outputs and v_1, \dots, v_s weights assigned to the inputs

1.CCR Model 2.BCC Model 3.Stochastic Model 4.The Additive Model 5.Russell Measure Models 6.Multiplicative models 7.Non- Controllable Variable (NCN) Model 8.Bounded Variable (BND) Model 9.Super Efficiency Model 10.Chance-Constrained DEA Model

Returns to Scale Models

The Constant Return to Scale assumption is only appropriate when all DMU's are operating at an optimal scale. Imperfect competition, constraints on cost, etc may cause a DMU to be not operating at optimal scale. Banker, Charnes and Cooper (1984) suggested an extension of the CRS DEA model to account for Variable Returns to Scale (VRS). The use of the CRS specification when not all DMU's are operating at the optimal scale, will result in measures of Technical Efficiency (TE) which are confounded by Scale Efficiencies (SE). The use of the VRS specification will permit the calculation of TE devoid of the SE effect. The CRS linear programming problem can be easily modified to account for VRS by adding the convexity constraint.

Chance Constrained Model

Chance constrained programming developed by Charnes et al.(1963) is an operations research approach for optimization under uncertainty when some or all coefficients in a linear program are random variables distributed in accordance with some probability law.. Chance constrained programming provides an approach for decomposition of the total variation in the data for each DMU in to its two components of noise and inefficiency based upon an envelopment procedure of the confidence regions. The confidence regions $D_j(\gamma)$ at probability level γ for each DMU_j, $j=1, \dots, 3$ can be determined by the panel data. Assuming joint normality, the chance constrained which generates a supporting hyper plane to a confidence region in R^n at probability level γ is the one generated by the probability level $\Omega = \Phi(c)$ where $P(\chi^2 \leq c^2) = \gamma$, $\Phi(c)$ is the standard normal distribution function, and χ^2_f is distributed χ^2 with f degrees of freedom. The CC-model provides an approach to incorporate the influence of a larger or smaller variation in activity data in efficiency evaluations as the sensitivity of efficiency scores can be analyzed by solving the model relative to varying specified levels of stochastic

noise

Chance Constrained Formulation

The chance constrained formulation of system proposed by Land et al. is as shown below.

$$\text{Min } (\theta, \lambda) \theta, \text{ subject to } P(Y \lambda \leq y_0) \leq \alpha, X \lambda \leq \theta x, \lambda \geq 0$$

The chance constraint $P(Y \lambda \leq y_0) \leq \alpha$ shows that only α or less than α percent of DMUs will produce output exceeding the best practice level. In the p-model approach, it is the probability of efficient input-output combinations from the evaluated DMU that is maximized under a set of chance constraints. Given below is the p-model CCDEA formulation that extends the CCR (ratio) model of DEA.

$$\text{Max } (v, u) P \left\{ \sum_{r=1, \neq 0}^s u_r y_{ro} / \sum_{j=1, \neq 0}^n v_j x_{jo} \geq 1 \right\} \text{ subject to}$$

$$P \left\{ \sum_{r=1, \neq 0}^s u_r y_{rj} / \sum_{j=1, \neq 0}^n v_j x_{ij} \leq 1 \right\} \geq 1 - \alpha_j \quad j=1, 2, \dots, n, u_r, v_i \geq 0, \forall r, i.$$

Here 'P' means probability and the outputs and inputs are identified as random variables with a known probability distribution. While $0 \leq \alpha_j \leq 1$ is scalar, specified in advance, which represents the allowable chance (risk) of failing to satisfy the constraints with which it is associated.

The weights for that DMU are the individual components in the vector λ , which are the solution to the program

$$\text{Min } \theta \text{ subject to } P(Y^n \lambda \leq y_{n0}) \leq 0.05, n=1, 2, \dots, N, \theta x_{m0} \geq X^m \lambda, m=1, 2, \dots, M,$$

$$\theta \text{ unrestricted in sign}, \lambda \geq 0$$

First, we assume that all observed outputs coincide with their mathematical expectations.

Also assume that all outputs are stochastically independent; the performance at one

school site is independent of that another site. Third, we assume that the within school

variability of each output (as measured by the variance) is the same for all outputs and at

all school sites. Mathematically $E y_{ni} = y_{ni}$ for all n and i , $\text{Cov}(y_{ni}, y_{nj}) = 0$ for all n and for

$$i \neq j, \text{Var } y_{ni} = \text{constant and equal for each } n, \text{ say } c^2$$

Under the simplifying assumptions, the chance constrained DEA program can be written

$$\text{Min } \theta \text{ subject to } Y^n \lambda - y_{n0} - 1.645 \sigma \geq 0, n=1, 2, \dots, N, \theta x_{m0} - X^m \lambda \geq 0, m=1, 2, \dots, M,$$

$$\theta \text{ unrestricted in sign}, \lambda \geq 0 \text{ with } \sigma = c \left(\sum (\lambda^2 - 2\lambda_0 + 1) \right)^{0.5}$$

Compared with the conventional deterministic DEA formulation, this chance constrained formulation requires information about one additional parameter, namely the constant c .

Inspecting the data, it is found that

1. The chance constrained DEA efficiency ratios are higher than the deterministic ones.
2. The greater the stochastic variability of outputs (the greater the coefficient c), the greater is the band of output territory that is permitted outside the envelope. The envelop moves successively closer to any given observation and the efficiency ratio approaches one.
3. The relative rankings of schools with efficiency scores less than one differ somewhat in the chance- constrained analysis as compared to deterministic DE

Efficiency Analysis -1

The project is studied at three stages with the assistance of three Post graduate students, doing their MSc Dissertation. At first stage 11 Schools of Thrissur Urban area is selected for the collection of data. With 3 Government Schools, 5 Private Un Aided Schools, 3 Private Aided Schools Sample frame is formed and samples were collected from one out of 10 students of each class of 7th, 9th and 11th standard students of this Schools. The questionnaire having 20 questions is used to appraise their school performance . Questions for Students and Parents were prepared in 5 point scale. Average responses and standard deviation of the data is used for the preparation of input and outputs for each Schools called DMUs

The following are the designed inputs and outputs for each DMU –School – for applying DEA

Inputs: 1. Percentage of available Physical Facilities (PPF) 2. Percentage of available Ancillary Facilities (PAF) 3 .Teacher's Qualification Index (TQI) 4. Education-Occupation Index of parents (EOI)

Outputs: 1. Students' Satisfaction (SS) 2. Students' participation in Extra-curricular Activities (SEA) 3. Social Awareness in Students (SAS)

.Applying Frontier Analysis, the relative efficiency of High School educated parents compared Postgraduate parents with respect to Students Satisfaction is 14% and 35% respectively.

School efficiency comparison using DEA

Considered inputs were PPF, PAF and EOI for 11 schools and output was the lone SS finalized after confirmatory factor analysis.

By the BCC model LPP for G1 is: $\langle G1 \rangle \quad \text{Mini } \theta_B$

Sub to $1.83 \theta_B - 1.83\lambda_{G1} - 1.90\lambda_{G2} - 1.44\lambda_{G3} - 1.70\lambda_{A1} - 1.80\lambda_{A2} - 1.57\lambda_{A3} - 1.51\lambda_{U1} - 1.54\lambda_{U2} - 1.86\lambda_{U3} - 1.60\lambda_{U4} - 1.55\lambda_{U5} \geq 0$

$1.99 \theta_B - 1.99\lambda_{G1} - 1.81\lambda_{G2} - 1.79\lambda_{G3} - 1.77\lambda_{A1} - 1.88\lambda_{A2} - 1.80\lambda_{A3} - 1.96\lambda_{U1} - 1.86\lambda_{U2} - 1.81\lambda_{U3} - 1.62\lambda_{U4} - 1.69\lambda_{U5} \geq 0$

$2.26 \theta_B - 2.26\lambda_{G1} - 2.20\lambda_{G2} - 2.82\lambda_{G3} - 3.92\lambda_{A1} - 3.54\lambda_{A2} - 3.82\lambda_{A3} - 3.32\lambda_{U1} - 3.97\lambda_{U2} - 3.45\lambda_{U3} - 4.19\lambda_{U4} - 3.97\lambda_{U5} \geq 0$

$$1.84\lambda_{G1}+1.83\lambda_{G2}+1.51\lambda_{G3}+1.79\lambda_{A1}+1.91\lambda_{A2}+1.82\lambda_{A3}+1.83\lambda_{U1}+1.95\lambda_{U2}+1.81\lambda_{U3}+1.77\lambda_{U4}+1.74\lambda_{U5} \geq 1.84$$

$$e (\lambda_{G1}+\lambda_{G2}+\lambda_{G3}+\lambda_{A1}+\lambda_{A2}+\lambda_{A3}+\lambda_{U1}+\lambda_{U2}+\lambda_{U3}+\lambda_{U4}+\lambda_{U5}) = 1, \quad \lambda \geq 0, \text{ where } \theta_B \text{ is a scalar.}$$

Similar LPPs are formulated for rest of the DMUs. Solving this problem gives the optimum solution for θ_B and also the corresponding values of the parameters (λ^* , s^- , s^+), where s^+ and s^- are corresponding slack and surplus variables. For an inefficient DMU there exists a reference set, E_0 based on λ^* such that $E_0 = [j / \lambda_j^* > 0]; j \in 1, 2, \dots, n$.

The DMU without any excess of inputs and shortfalls of outputs is considered as efficient so that 7 DMUs are efficient –All Govt and all but one unaided schools are efficient. But all aided schools are inefficient. The removal of inefficiency is achieved by reducing the inputs PPF, PAF, EOI by 8.37%, 2.59% and 2.59% respectively . In fact, based on the reference sets and λ^* , the input and output values can be modified to bring A2 in to efficient status. One Unaided (specifiable) is the optimum efficient model with respect to minimum input and the next three are Govt schools followed by three Unaided Schools. Three Aided Schools and one Unaided school lack of efficiency from 1% to 5%. It is observed that out of 11 DMUs 7 are efficient and 4 are inefficient. ie, Aided schools are inefficient compared to Government and Unaided.

Efficiency Analysis -2

Second stage of study is associated with 40 schools of Thrissur Corporation Area and nearby Panchayaths, and the data is collected by the Post graduate students of Department of Statistics, St. Thomas' College Thrissur. Only one questionnaire having 24 questions is used at this stage and students were selected by stratified sampling method taking 5 to 10 students from each class. Stratification is done considering the performance and socio-economic standards of students. 10 Government Schools, 27 Private Aided schools and 3 Unaided school were considered to collect information and 800 samples were collected. As a part of this project students of 40 independent DMUs with 4 inputs and 2 outputs were considered. The following Inputs/Outputs are framed with respect to the 24 questions.

A. School Related Variables based on facilities in the school

(1) Available Teaching Facilities (ATF) -- availability of library, character formulation, teaching methods, parent teacher communication. (2) Available Ancillary Facilities (AAF)- the availability of learning , facilities, club, stipend.

B. Home Related Variables (3) Parents Contribution (PC) (4) Parents Education Occupation Index (EOI)

C. Studied Outputs (5) Students Satisfaction (SS) (6) Students Extra-curricular Activities (SEA)

1) ATF 2) AAF 3) PC 4) EOI are taken as input and 1) SS 2) SEA as outputs

Considering various aspects as the input and output, sample size, data score, dependence and independence, return to scaling etc, BCCI is the suitable model used for performance analysis and the following results were obtained

The Schools reaching maximum score Govt- G1,G2,G6,G7,G8,G9 (6/10 =60%) and Aided A1,A2,A3,A4, A5, A10, A12,A14, A15, A16 ,A17 (11/27=41%).

No Excess or shortage is seen in 11 aided and 6 Govt schools and score is reached unity ,but 3 Aided schools haven't reached a score 1 but no shortage or excess is found. Excess in ATF is found in 4 Aided and one Govt school. Excess in AAF is found in 2 Aided and 2 Govt and one Unaided school .Excess in PC is found in 6 Aided and 3 Govt and one Unaided school .Excess in AAF is found in 2 Aided and 2 Govt and one Unaided school .Excess in EOI is found in 5 Aided and 1 Govt and 2 Unaided school. Shortage in SS is found in 3 Aided and 2 Govt and 2 Unaided school. Shortage in SEA is found in 1 Aided school.

BCCI projection is achieved by

$$x^{\wedge} G10 \leftarrow \theta * x ATF = 0.9640 * 2.22 (-11.95\% \text{ reduction})$$

$$x^{\wedge} G10 \leftarrow \theta * x AAF = 0.9640 * 2.43 (-5.46\% \text{ reduction})$$

$$x^{\wedge} G10 \leftarrow \theta * x PC = 0.9640 * 2.4964 (-31.08\% \text{ reduction})$$

$$x^{\wedge} G10 \leftarrow \theta * x EOI = 0.9640 * 2.184 (-3.6\% \text{ reduction})$$

$$y^{\wedge} G10 \leftarrow s^{+*} + y SS = 1.81 + 0.049 (4.38\% \text{ reduction})$$

In this case , it is observed that in 40 DMUs 17 are efficient.

Return to Scale become constant in 6 of the 17 efficient DMUs of which 3 are Govt Schools and increasing in 34 DMUs. Reference set indicates the modification needed by the schools with respect to efficient schools to reach optimum. One Unaided school require further improvement with respect to 3 Aided schools and one Govt school. Least performing Unaided School required changes with respect to 1 Govt and 2 Aided schools and other one Aided school is in need of modification with respect to 3 Govt Schools and one Aided school.

Projection is less than the scores of all input and high in SS for 2 Govt and 4 Aided school. All the inputs are low in projection for 11 Aided, 3 Govt and 2 Unaided institutions.

Efficiency Analysis -3

In the third phase of performing efficiency analysis of Government, Aided and Unaided schools using DEA, response of the parents and students of 7th, 9th & 11th standard students is considered. 36 independent DMUs (18 Schools-Appendix 3) with 7 input and 4 outputs are applied in the BCCI model. In another case, students, parents and teachers of 11 institutions containing 3 Aided, 5 Unaided and 3 Government Schools were considered, with 8 inputs and 3 outputs.

School related variables

1) Percentage of Teaching Facilities (PTF) 2) Percentage of Physical Facilities (PPF) Percentage of Ancillary Facilities (PAF) 4) Teacher Qualification Index (TQI) 5) Teacher Experience Index (TEI) 6) Number of Students Per Teacher (SPT)

Home related variable

1) Parent's Education Occupation Index (EOI)

Studied outputs

1) Students Satisfaction (SS) 2) Social Awareness of Students (SAS) 3) Students Extra-curricular Activities (SEA).

The main objective of this study is to find performance of various schools based on opinion of students and parents forming 18 DMUs on students and 18 DMUs on Parents of 18 schools (Appendix-3) with respect to 7 inputs and 3 outputs.

Corresponding to each DMU, formulate LPP in the form (BCC_I)

$\text{Min}_{\theta_B, \lambda} \theta_B$ subject to $\theta_B x_o - X\lambda - s^-$, $Y\lambda + s^+ = y_o$, $e\lambda = 1$, $\lambda, s^+, s^- \geq 0$,

Where θ_B is a scalar.

.Thus it is observed that out of 36 DMUs, 17 are efficient and 19 are inefficient.

Now Consider the same model with 5 input and 2 output excepting SPT, EOI and SEA.

The RTS shows that only one efficient DMU is increasing and the remaining 5 efficient DMUs are constant. From this analysis it is observed that out of the 11 DMUs, 6 DMUs (Schools) are efficient (1 govt, 2 aided with respect to opinion of parents and 2 govt and 1 aided with respect to the opinion of students) the remaining 5 DMUs are inefficient by the opinion of both parents and students

Efficiency Analysis - Chance Constrained (CC) DEA Model

. As a part of this project, consider the chance constrained programming models that allow changes the orientation of DEA. Instead of treating only data generated from past behaviour, the orientation is now directed to anticipation. The modern theory of DEA has its roots in the mathematical theory of production. The core of this theory is deterministic. This paper casts a wider net, aiming for a bigger catch: the possible presence of stochastic variability in the production relationships, and the representation of production frontiers by chance- constrained programming.

The chance- constrained model has been developed subject to the requirement that the production possibility set satisfies a number of maintained hypotheses. It is important to be aware that a number of additional assumptions have been employed in the specification of the model.

1. A parametric specification of the distribution of the random vectors (Y_j, X_j) , $j=1,2,\dots,N$ is required.
2. The estimation of the mean vector and the covariance matrix for each DMU requires a set of panel data.
3. The model allows for assigning a varying fraction of the variation in data to noise for each of the N DMUs.

It is equally important to be aware that the assumptions above relate to the specification rather than the foundation of the CC-model, a number of the listed assumptions may be relaxed or subjected to a specification.

The following are the inputs and outputs framed

Input (I) 1) Performance Score- PS based on questions defining qualities reached by the students during the course.

Output (O) 1) Students Score SS based on Examination results 2) Teachers Score TS based on teaching skill, experience, training.3) Environment Score ES based on infrastructure and convenience

Here there are 19 DMUs with one input and 3 outputs taken from 5 Govt, 5 Unaided and 9 Aided school.

In order to carry out the calculations of chance constrained DEA we need information about the joint probability distribution of all random variables. Such information can, in principle, be inferred from the frequency distributions of the underlying data. The only data available to us are the average test and scale scores for each of the output variables.

Standard distribution theory implies that the average scores for each output also are normally distributed. First, assume that all observed outputs coincide with their mathematical expectations. Second, all outputs are stochastically independent. Third, the variability of each output is the same for all the outputs.

Under our simplifying assumptions, the chance- constrained DEA program can be written,

$$\text{Min}_{\theta, \lambda} \theta, \text{ subject to } P(Y\lambda \leq y_0) \leq \alpha, X\lambda \leq \theta x, \lambda \geq 0$$

Also BCC (I) Model is applied as to study the efficiency of schools by both models

LPP in the form (BCC_I) is

$$\text{Min}_{\theta_B, \lambda} \theta_B \text{ subject to } \theta_B x_0 - X\lambda - s^- Y, \lambda + s^+ = y_0, e\lambda = 1, \lambda, s^+, s^- \geq 0 \text{ where } \theta_B \text{ is a scalar.}$$

The chance constraint $P(Y\lambda \leq y_0) \leq \alpha$ shows that only α or less than α percent of DMUs will produce output exceeding the best practice level. For inefficient DMU with respect to SS, TS and ES

A2, A6, A8 in BCC model. TS also had no shortage by CC and A2, A7, U1 in BCC model. 1 Govt, 5 Aided and all Unaided schools lacks in environmental satisfaction in CC model and it is 2 Govt, 3 Aided and 4 Unaided schools by BCC model.

Thus little more accurate weights can be attained by CC Model so that efficiency reaching DMUs and other DMUs attaining Technical Efficiency by the projection is also more efficient. Based on this reference set and λ^* we can express the input and output values needed to bring G3 in to efficient status as

$$0.8315 \times (\text{output of TS}) = 0.6591 \times (\text{output TS of G4}) + 0.1165 \times (\text{output TS of G5}) + 0.2242 \times (\text{output of TS of A4}) + 0.1637$$

The projection table contains projections of each DMU on to the efficient frontier by the chosen model. The CC and BCC model projection is achieved by,

$$\hat{x}_0 = \theta^* x_0 - s^{-*} = \sum_j x_j \lambda_j^*, j \text{ belong to } E_0$$

$$\hat{y}_0 = y_0 + s^{+*} = \sum_j y_j \lambda_j^*, j \text{ belong to } E_0$$

From this analysis using CC model it is found that out of the 19 DMUs, 5 DMUs are efficient and the remaining 14 DMUs are inefficient. Among the efficient Schools 3 are Govt and 2 are Aided Schools.

From the analysis using BCC model it is found that out of the 19 DMUs, 6 DMUs are efficient and the remaining 13 DMUs are inefficient. Among the efficient Schools 3 are

Govt and 3 are Aided School

Summary

Table :3.12 Summary on inputs and outputs based on BCC model CC model

DEA model = CC and BCC			
Problem = DMU SCHOOLS			
No. of DMUs = 19			
No. of Input items = 1			
Input(1) = PS			
No. of Output items = 3			
Output(1) = SS , Output(2) = TS , Output(3) = ES			
CC Model		BCC Model	
Returns to Scale			
Constant ($0 \leq \text{Sum of Lambda} < \text{Infinity}$)		Variable (Sum of Lambda = 1)	
Frequency in Reference Set			
Peer set	Frequency to other DMUs	Peer set	Frequency to other DMUs
G1	2	G1	1
G4	5	G4	9
G5	7	G5	7
A4	14	A4	8
A9	2	A5	2
		A9	4
No. of efficient DMUs = 5		No. of efficient DMUs = 6	
No. of inefficient DMUs = 14		No. of inefficient DMUs = 13	

Conclusion

1. Frontier Analysis shows high relative efficiency exists in the students with Post Graduate Parents
2. With respect to three inputs –PPF, PAF , EQI and one Output SS, applying BCC model ,7 schools are found to be efficient. One unaided school is highest ranked efficient school followed by 3 Govt Schools and then 3 Unaided schools.
3. 4 Govt Schools were referenced 6 times and 4 unaided schools 8 times for reaching better performance of other schools
4. Most of the combinations of inputs and outputs give the better performing units are Govt Schools followed by unaided schools.

5. Based on 4 inputs-ATF, AAF,PC,EOI and 2 output SS and SEA and BCC model on 40 schools provide 17 efficient schools of which 6 are govt schools out of 10 govt schools(60%). Also 11 Aided schools out of 27 (41%) are efficient. The best ranked is Aided and then 2 are Govt Schools
6. Out of 23 inefficient schools, one Govt school can be made efficient referencing to 1 govt school and 1 aided school , another aided school is referenced by 2 govt schools and third Govt school is referenced by 2 govt & 1 aided school.
7. Return to Scale become constant in 6 of the 17 efficient DMUs of which 3 are Govt Schools and it increases in 34 DMUs.
8. With respect to 18x2 DMUs (Parents and Students of 18 Schools) depending of 7 inputs and 3 outputs ,it is found that 15 schools were totally efficient while parents of 1 aided and govt schools were inefficient and students of one aided school is also inefficient
9. Out of the 11 DMUs, with 3 input and one output, 6 DMUs (Schools) are efficient. (1 Govt & 2 Aided school with respect to opinion of parents and 2 Govt and 1 Aided school with respect to the opinion of students.) .One school is only reaching CRS while 5 schools are VRS increasing.
10. Using one input and three output (input PS, output SS,TS,ES) Chance constrained model shows 5 efficient schools but BCC I model gives 6 schools
11. Among the 5 schools 3 were Govt schools and 2 Aided schools. In the BCCI model one more Aided school is efficient.
12. Slack is minimal in CC model while it is more in BCC Model
13. 12 schools is referenced by one govt and one aided school to reach efficiency and 2 schools were made optimum efficiency by referencing with 1 Govt and 2 Aided school by CC model
14. For BCC model , in the inefficient schools ,3 aided schools become efficient by improving with respect to 1 govt & 1 aided school or 2 aided schools.10 schools is referenced by 1 govt & 1 aided school and 1 school by 2 govt & 2 aided school. All the 5 unaided schools were needed reference of 1 govt and 1 aided school.

15. No difference in projection is found in the factor TS in both model while the factor SS shows Difference in one aided school.

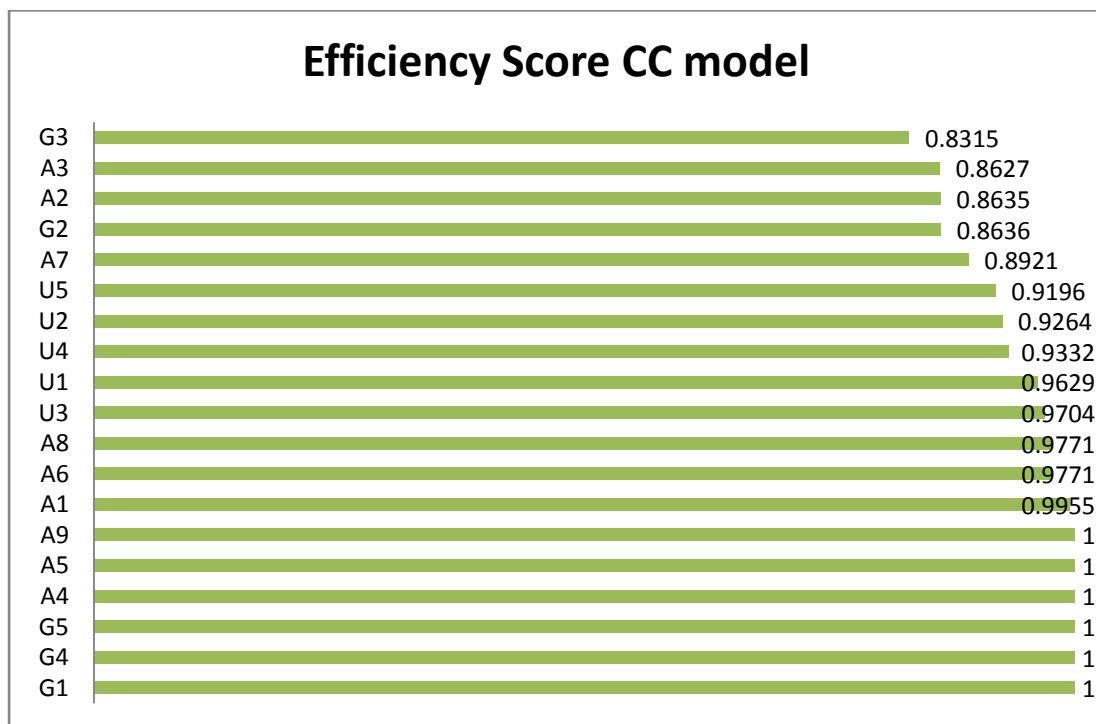
16. In CC model out of 30 references of efficient models 14 were govt and 16 were Aided. In BCC model out of 31 references of efficient models 17 were govt and 14 were Aided

17. CC model reaches CRS while BCC holds CRS only in 4 schools of which 3 are efficient. VRS is found in 15 schools and only 5 are efficient

18. Among the Govt schools 60% are efficient and in Aided 22% is efficient in CC model and 33% in BCC model.

19. Overall study on performance of schools shows higher efficiency for Govt schools (about 50-60%) and then Aided School (30—40%)

20. Parents also support the efficiency of Govt Schools. Environmental as well as convenience of other schools are not efficient as expected.



References

1. Kenneth C. Land, C.A. Knox Lovell and Sten Thore (1993). *Chance-Constrained Data Envelopment Analysis*, John Wiley and Sons .
2. O.B. Olesen and N.C. Petersen (1995). *Chance Constrained Efficiency Evaluation*, INFORMS.
3. Jaume Puig-Junoy (1998). *Technical Efficiency in the Clinical Management of Critically ill Patients*, John Wiley and Sons Ltd .
4. O.B. Olesen, N.C. Petersen (1999). *Probabilistic bounds on the virtual multipliers in Data Envelopment Analysis: Polyhedral Cone Constraints*, Kluwer Academic Publishers, Boston.
5. Hiroshi Morita and Lawrence M. Seiford (1999). *Characteristics on Stochastic Data Envelopment Analysis Efficiency -Reliability and Probability being efficient*, Journal of the Operations Research Society, Japan .
6. William W. Cooper and Honghui Deng (2001). *Chance Constrained Programming Approaches to Technical efficiencies and inefficiencies in Stochastic Data Envelopment Analysis* .
7. Tser- Yieth Chen (2002). *A comparison of Chance- Con strained Data Envelop-ment Analysis and Stochastic Frontier Analysis: Bank Efficiency in Taiwan*, Journal of Operational Research Society.
8. R. Ramanathan (2003). *An Introduction to Data Envelopment Analysis- A Tool for Performance Measurement*, Sage Publications .
9. William W. Cooper, H. Deng, Zhimin Huang, Susan X. Li (2004). *Chance- Constrained Programming Approaches to Congestion in Stochastic Data Envelopment Analysis*, European Journal of Operational Research.
10. O.B. Olesen (2004). *Comparing and Combining Two Approaches for Chance- Constrained Data Envelopment Analysis*.
11. Frantisek Brazdik (2004). *Oriented Stochastic Data Envelopment Analysis - Derivation and Application*.
12. Frantisek Brazdik (2005). *Oriented Stochastic Data Envelopment Analysis: Rank-ing Comparison to Stochastic Frontier Approach*.
13. S. Ramezanzadeh, M. Memariani and S. Saati (2005). *Data Envelopment Analysis with Fuzzy Random Inputs and Outputs: A Chance- Constrained Data Envelopment Analysis Approach*, Iranian Journal of Fuzzy Systems.

14. Adnan Kasman And Ervin Turgutlu (2007). A Comparison of Chance- Constrained Data Envelopment Analysis and Stochastic Frontier Analysis: An application to the Turkish Life Insurance Industry..
15. A. Udhayakumar, V. Charles, Mukesh Kumar (2010). Stochastic Simulation Based Genetic algorithm for Chance-Constrained Data Envelopment Analysis Problems, International Journal of Management Science .
16. M. Khodabakhshi (2010). Chance Constrained Additive Input Relaxation Model in Stochastic Data Envelopment Analysis, International Journal of Information and Systems Sciences.
17. O.B. Olesen and N.C. Petersen. Foundations of Chance-Constrained Data Envelopment Analysis for Pareto- Koopmann Efficient Production Possibility Sets .
18. Jibendu Kumar Mantri. Research Methodology on Data Envelopment Analysis, Universal- Publishers, Florida, USA.