FINAL REPORT OF
MINOR RESEARCH PROJECT
ON

NANOFLUID FLOW PAST AN INCLINED STRETCHING SHEET
WITH APPLICATIONS

SUBMITTED TO

THE UNIVERSITY GRANTS COMMISSION

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NANOFLUID FLOW PAST AN INCLINED STRETCHING SHEET WITH APPLICATIONS

CERTIFICATE

This is to certify that the minor research project entitled **"BOUNDARY LAYER FLOW OF NANOFLUID PAST AN INCLINED STRETCHING SHEET WITH APPLICATIONS"** submitted by Dr.Julie Andrews, Assistant Professor, Department of Mathematics is a genuine work done by her under the financial assistance and guidance of UGC and is here by submitted for approval.

Principal,

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NANOFLUID FLOW PAST AN INCLINED STRETCHING SHEET WITH APPLICATIONS

1. INTRODUCTION

Nanofluids, in which nano-sized particles (typically less than 100 nanometers) suspended in liquids, have emerged as a potential candidate for the design of heat transfer fluids. The main goal of nanofluids is to achieve the highest possible thermal properties at the smallest possible concentrations by uniform dispersion and stable suspension of nanoparticles in host fluids. Nanofluid is defined as a colloidal solvent containing dispersed nanometer-sized particles (~1-100 nm). Cooling becomes one of the top technical challenges facing high-tech industries such as microelectronics, transportation, manufacturing and metallurgy. Nanofluids, when used as coolants can provide dramatic improvements in the thermal properties of host fluids.

The term nanofluid was first coined by Choi where he described the future and hope of this application of nanotechnology. The emergence of nanofluids as a new field of nano scale heat transfer in liquids is related directly to miniaturization trends and nanotechnology. Nanofluids owe its history to the Advanced Fluids Program (AFP) at Argonne National Laboratory (ANL), U.S.A that encompassed a wide range (meters to nanometers) of size regimes and eventually the wide research road became narrow, starting with large scale and descending through micro scale to nano scale, culminating in the invention of nanofluids. New advances in producing nano-sized metallic or nonmetallic particles have allowed producing this new kind of fluid. Most notably, the nanofluid thermal conductivities were found to vary non-linearly as a function of the nanoparticle loading. Nanofluids due to their excellent properties including better stability and increased thermal conductivity have been investigated by several researchers.

Synthesis of nanofluids

Nanofluids do not occur in nature. Actually, nanofluids need to be synthesized in industrial laboratories employing various manufacturing methods and techniques. Stable and highly conductive nanofluids are produced by one-step and two-step production methods.

Production methods of nanofluids

The one-step (single-step) method simultaneously makes and disperses nanoparticles into base fluids. In **the two-step method**, the nanoparticles are made using one of the nanoparticle processing techniques and then are dispersed into base fluids. Most nanofluids containing oxide nanoparticles and carbon nanotubes reported in the open literature are produced by two-step process. Although the two-step method works well for oxide nanoparticles, it is not as effective for metal nanoparticles such as copper. For nanofluids containing high-conductivity metals, it is clear that the single-step technique is preferable to the two-step method.

In order to prepare nanofluids, various nanoparticles are to be produced and host fluids into which the nanoparticles are to be dispersed need to be available. Researchers have found out many materials that can be used as base fluids and nano particles.

Nanoparticle material types and host liquid types

Nanoparticles used in nanofluids have been made of various materials such as oxide ceramics like Aluminum Oxide (Al_2O_3), Cuprous Oxide (CuO), nitride ceramics such as Aluminum Nitride (AlN), Silicon Nitride (SiN), carbide ceramics such as Silicon Carbide (SiC), Titanium Carbide (TiC), metals copper (Cu), silver (Ag), gold (Au), semiconductors Titanium dioxide (TiO_2), Silicon Carbide (SiC), carbon nanotubes, and composite materials such as alloyed nanoparticles $Al_{70}Cu_{30}$ or nanoparticle core-polymer shell composites. In addition to the nonmetallic, metallic and other materials for nanoparticles, completely new materials and structures such as materials ‘doped’ with molecules in their solid liquid interface structure may also have desirable characteristics.

Many types of liquids like water, ethylene glycol and oil have been used as host liquids in nanofluids.

Nanoparticles have to be manufactured at first for the preparation of nanofluids. There are various methods available in industries, for the manufacture of nanoparticles.

Methods of manufacture of nanoparticles

Fabrication of nanoparticles can be classified into two broad categories:

1. Physical processes
2. Chemical processes

Currently a number of methods exist for the manufacture of nanoparticles.

Physical method includes Inert Gas Condensation (IGC) developed by Granquist and Buhrman (1976) and mechanical grinding. Chemical method includes Chemical Vapour Deposition (CVD), chemical precipitation, micro emulsions, thermal spray and spray pyrolysis.

The current processes for making metal nanoparticles include IGC, mechanical grinding, chemical precipitation, thermal spray and spray pyrolysis. Sonochemical method makes suspensions of iron nanoparticles stabilized by Oleic acid. Most recently, Chopkar et al. (2006) produced alloyed nanoparticles $Al_{70}Cu_{30}$ using ball milling. In ball milling, balls impart a lot of energy to a slurry of powder, and in most cases some chemicals are used to cause physical and chemical changes. These nanosized materials are most commonly produced in the form of powders. In powder form, nanoparticles are dispersed in aqueous or organic host liquids for specific applications.

Although nanoparticles often refer to spherical shapes, there are also various anisotropic shapes. These refer to all shapes other than spherical. Such shapes require more than one parameter to describe their shapes. The most common ones are nano rods and nano triangles. Several other shapes, such as tripods, tetra pods, stars, flowers and sheets are known, and in several cases synthetic flexibility does not exist. However, it is not possible to get most of these in the solution phase and so out of the discussion range.

2.APPLICATIONS OF NANOFLUIDS

Nanofluids find most of their applications in thermal management of industrial and consumer products as efficient cooling is vital for realizing the functions and long-term reliability of the same. There are a large number of tribological and medical applications for nanofluids. Recent studies have demonstrated the ability of nanofluids to improve the performance of real-world devices and systems such as automatic transmissions.

Nanofluids which are smart coolants are used for a number of applications in industry and technology. As smart coolants they could be used for a variety of choices. They come under heat transfer applications, automotive applications, electronic applications and biomedical applications.

Replacing water with the nanofluid can really create a very large energy profit in the industries which will benefit the economy of nations. Owing to their enhanced thermal properties they could be used in engineering applications from use in the automotive industry to the medical arena to use in power plant cooling systems and computers. The use of nanofluids in power plants are being considered well.

The earth's crust is a source of energy required for using in industries and technologies as a whole, but extracting geo thermal energy is extremely hot. In order to cool the big pipes used for the extraction of energy, nanofluids could be considered as the best option in the new trend. Nanofluid which is the fluid superconductor could be used for cooling the machinery in the drilling processes for geothermal extraction.

In using automotives, a large amount of heat is produced for the working of the sophisticated machines used. Nanofluids used reduce the heat energy and act as excellent coolants. In the field of aerodynamic designs, the use of nanofluids as coolants would allow for smaller size and better positioning of the radiators.

Nanofluids are used in fuels increase the quality of fuels for combustion engines. Heating, ventilation and air-conditioning (HVAC) could benefit from the high thermal conductivity offered by nanofluids that resulted from addition of nanoparticles.The biomedical applications of nanofluids include cancer treatment and cryosurgery and so on.

a) Cooling applications

The cooling applications of nanofluids include Crystal Silicon Mirror Cooling, Electronics Cooling, Vehicle Cooling, Transformer Cooling, Space and Nuclear systems Cooling, Defense applications and so on. The benefits of using nanofluids as a room-temperature coolant are clear, including dramatic enhancement of cooling rates while operating the advanced cooling system at room temperature. Moreover, the possibility of thermal distortion and flow-induced vibration will be eliminated by passing the nanofluids through micro channels within the silicon mirror itself.

b) Tribological Applications

Nanofluid technology can surely help to develop better oils and lubricants. Recently the use of nanoparticles in lubricants to enhance tribological properties of lubricants, such as load-carrying capacity, antiwear and friction-reducing properties between moving mechanical components has developed as the nanofluid research activity. Further, in lubrication application, it has been reported that surface-modified nanoparticles stably dispersed in mineral oils are very effective in reducing wear and tear and enhancing load-carrying capacity. Nanoparticles decrease friction remarkably on the surface. In short, as nanofluids reduce friction and wear, they could be used in numerous applications for oil and gas drilling.

c) Biomedical Applications

Nanofluids can be formulated for a variety of uses for faster cooling. Nanofluids are now being developed for medical applications, including cancer therapy. As traditional cancer treatment methods have significant side effects, Iron-based nanoparticles can be used as delivery vehicles for drugs or radiation without damaging nearby healthy tissue by guiding the particles up the bloodstream to a tumour with magnets. Moreover, nanofluids could be used for safer surgery by cooling around the surgical region, thereby enhancing a patient’s chance of survival and reducing the risk of organ damage. In contrast to cooling, nanofluids could be used to produce higher temperatures around tumours; to kill cancerous cells without affecting nearby healthy cells.

d) Other potential applications

Though the two salient areas of nanofluid applications are that of electronics and engine cooling, other possible areas for application of nanofluids technology include cooling a new class of super powerful and small computers and other electronic devices for use in military systems, airplanes or spacecraft as well as for large-scale cooling. Actually, nanofluids could be made use of to maintain a high temperature gradient in thermo electronics that would convert waste heat to useful electrical energy. In buildings, nanofluids could increase energy efficiency without the need to use a powerful pump, thereby saving energy in a HVAC (Heating, Ventilation and Air-Conditioning) system and providing major environmental benefits. In the renewal energy industry, nanofluids could be utilized to enhance heat transfer solar collections to storage tanks and to increase the energy density. To all of this, it must be added that nanofluids could be used for cooling for major process industries, including materials, chemical, food and drink, oil and gas, paper and printing and textiles.

The novel projected applications of nanofluids include sensors and diagnostics that instantly detect chemical warfare agent in water or water or food borne contamination; biomedical applications include cooling medical devices, deleting unhealthy substances in the blood; and development of advanced technologies such as advanced vapour compression refrigeration systems. It is clear that nanofluids will be increasingly important for high-value added niche applications as well as for high-volume applications.

3.THE VARIATION IN VALUES FOR DIFFERENT TYPES OF NANOFLUIDS

The values of densities of Copper-water nanofluid and alumina water nanofluid are found out using the following values in the table.

Table 1. The values of density of base fluid water, copper, alumina at 20° C (293 K)

	ρ (Kg/m^3)
Water	1000.52
Copper	8954
Alumina	3970

As the volume fraction ϕ is changed, there are variations in the values of density for different types of nanofluids. The values of density of copper-water nanofluid, alumina water nanofluid are as follows:

Table 2. The values of densities of different types of nanofluids

Value of ϕ	Copper-water nanofluid(Kg/m^3)	Alumina-water nanofluid(Kg/m^3)
0.00	1000.52	1000.52
0.01	1080.05	1031.22
0.02	1159.59	1059.91
0.03	1239.12	1089.60
0.04	1318.65	1119.30
0.05	1398.19	1148.99
0.06	1477.72	1178.99
0.07	1557.26	1208.38
0.08	1636.79	1238.08
0.09	1716.33	1267.77

The effective density of the nanofluid is given by

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s$$

4. NANOFLUID FLOW OVER AN INCLINED STRETCHING SHEET

There have been many studies regarding heat transfer of fluids over stretching sheets till date as they have many practical applications in manufacturing and industries. In view of these, the present work is dealt with mixed convection boundary layer flow of a nanofluid past an inclined stretching sheet. The type of nanofluid considered for study under case is copper water nanofluid.

The present problem considers the steady, nonlinear two-dimensional, mixed convective boundary layer flow of a viscous, incompressible nanofluid past an inclined stretching surface with an angle of inclination α with the horizontal and stretching with the linear velocity $u_w(x) = ax$, where a is a constant and x is the coordinate measured along the stretching surface. The sheet is stretched keeping the origin fixed when α is 0° and the variations are made accordingly with the change in inclination angles. The Cartesian coordinates (x, y) are chosen such that x-axis is chosen along the stretching sheet and y-axis is chosen perpendicular to it. It is assumed that at the stretching surface, the temperature T and the nanoparticle fraction C take constant values T_w and C_w, respectively. The ambient values, attained as y tends to infinity, of T and C are denoted by T_∞ and C_∞, respectively.

Copper-water nanofluid

Water with Pr = 7.02 (considering base temperature as 293 K) is the base fluid considered for the present study. Copper-water nanofluid used for the present work is formed by mixing solid spherical copper nano particles of 100 nm diameter with water.

The effective density of the nanofluid is given by

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s$$

Also the specific heat capacity of the nanofluid is given by the following formula:

$$\left(c_p\right)_{nf} = (1-\phi)(c_p)_f + \phi(c_p)_s$$

as given by Santra et al. , where ρ_f and ρ_s are density, $(c_p)_f$ and $(c_p)_s$ are specific heat capacities of the base fluid and solid particle respectively.

The following equation as given by Brinkman is used to calculate the dynamic viscosity of the nanofluid:

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}$$

where μ_f gives the dynamic viscosity of the base fluid (μ_f = 0.001002) and ϕ is the solid volume fraction.

Also, the effective thermal conductivity of the nanofluid is determined by the model proposed by Patel et al. (2005) and the thermal conductivity k_{eff} for the two component entity of spherical particle suspension, is given by

$$\frac{k_{eff}}{k_f} = 1 + \frac{k_p A_p}{k_f A_f} + \frac{c k_p Pe A_p}{k_f A_f}$$

Here, we find

$$\frac{A_p}{A_f} = \frac{d_f}{d_p} \frac{\phi}{(1-\phi)}$$

and

$$Pe = \frac{u_p d_p}{\alpha_f}$$

where u_p is the Brownian motion velocity of the particle which is given by

$$u_p = \frac{2T k_b}{\pi \mu_f d_p^2}$$

where k_b is the Boltzmann constant ($k_b = 1.3806 \times 10^{-24}$ J/K) and T represents the temperature. Pe is the Peclet number. Here A_p and A_f are taken as the heat transfer areas of the paricle and the fluid respectively. The thermal conductivities of the solid particle and the base fluid are represented by k_p and k_f respectively. Here k_{eff} denotes k_{nf}. Also, d_p is the particle diameter and d_f is the molecular size of the liquid. Here the liquid particle size is taken as the molecular size of the liquid and for water, it is 2 Angstroms (2A°) and α_f is the thermal diffusivity of the fluid. The value of the constant 'c' is taken as 36000 which is taken from the available experimental data by Xuan and Roetzel (2000). The physical properties such as density, specific heat and thermal conductivity of the base fluid water, copper and alumina at 20°C (293 K) which are used for calculating the density, specific heat and thermal conductivity of copper-water nanofluid are given in Table 3 below.

Table 3 Physical properties of base fluid water, copper at 20° C (293 K)

	ρ (Kg /m^3)	c$_p$ (J /Kg.K)	k (W /m.K)
Water	1000.52	4181.8	0.597
Copper	8954	383.1	386

Governing equations of the flow

The basic steady equations for conservation of mass, momentum, thermal energy and concentration of nanoparticles for nanofluids considering the effects of Brownian motion and thermophoresis can be written in Cartesian coordinates x and y as given by Kuznetsov and Nield (2009; 2010). The Boussinesq approximation is employed and the four field equations embody the conservation of total mass, momentum, thermal energy and nanoparticles, respectively. With an assumption that the nanoparticle concentration is dilute, with a suitable choice for the reference pressure and using the standard boundary layer approximation, the governing boundary layer equations are given in the following form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}= g\beta_{nf}\,(T-T_\infty)\sin\alpha+g\,\beta^*_{nf}\,(C-C_\infty)\sin\alpha -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial x}+\nu_{nf}\left(\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$0= g\beta_{nf}\,(T-T_\infty)\cos\alpha+g\,\beta^*_{nf}\,(C-C_\infty)\cos\alpha -\frac{1}{\rho_{nf}}\frac{\partial p}{\partial y} \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}= \alpha_{nf}\left(\frac{\partial^{2}T}{\partial y^{2}}\right) + \varepsilon\left[D_B\left(\frac{\partial C}{\partial y}\,\frac{\partial T}{\partial y}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^{2}\right] \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^{2}C}{\partial y^{2}}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^{2}T}{\partial y^{2}}\right) \tag{5}$$

Moreover, the boundary conditions for the velocity and temperature of this problem are:

$$\begin{aligned} &u=u_w(x)=ax\ ,v=0\ ,T=T_{W\,,}\ C=C_W \quad at\ y=0\\ &u=0\ ,T=T_\infty\ ,C=C_\infty\ \ as\ y\to\infty \end{aligned} \tag{6}$$

Here, u and v are the velocity components in x and y directions respectively, p is the perturbed fluid pressure, T is taken as the temperature, ν_{nf} is the kinematic viscosity of the nanofluid, k_{nf} is the thermal conductivity, $(c_p)_{nf}$ is the specific heat, α_{nf} is the thermal diffusivity, β_{nf} is the coefficient of thermal expansion of the nanofluid, β^*_{nf} is the coefficient of volumetric coefficient of expansion with concentration, C is the nanoparticle volume fraction and ρ_{nf} is the density of the nanofluid respectively. Here, a is a positive constant, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient and $\varepsilon=\frac{(\rho c_p)_p}{(\rho c_p)_f}$ is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid with ρ being the density. The governing Prandtl boundary layer equations for the steady two dimensional laminar nanofluid flow over an inclined stretching surface when 0° ≤ α ≤ 60° take the following form:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{7}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu_{nf}\frac{\partial^{2}u}{\partial y^{2}}+g\,\beta_{nf}(T-T_\infty)\sin\alpha+g\,\beta^*_{nf}(C-C_\infty)\sin\alpha \tag{8}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}= \alpha_{nf}\left(\frac{\partial^{2}T}{\partial y^{2}}\right) + \varepsilon\left[D_B\left(\frac{\partial C}{\partial y}\,\frac{\partial T}{\partial y}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^{2}\right]$$

(9)

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_B\left(\frac{\partial^2 C}{\partial y^2}\right)+\frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right) \qquad (10)$$

METHOD OF SOLUTION

The following dimensionless variables are introduced in order to seek the solution of the problem:

$$\psi(x,y)=(a\nu_{nf})^{1/2}\,x\,f(\eta)\ ,\ \eta=y\left(\frac{a}{\nu_{nf}}\right)^{1/2}, \qquad (11)$$

$$\theta=\frac{T-T_\infty}{T_w-T_\infty}\ ,\qquad \phi=\frac{C-C_\infty}{C_W-C_\infty}$$

where $\psi(x,y)$ is the stream function and velocity components can be expressed as

$$u=u_w f'(\eta) \qquad (12)$$

$$v=-\left(a\nu_{nf}\right)^{1/2} f(\eta) \qquad (13)$$

Accordingly, momentum, energy and concentration equations together with the boundary conditions, can be written as

$$f'''+ff''-(f')^2+\lambda\theta\sin\alpha+\lambda^*\phi\sin\alpha=0 \qquad (14)$$

$$\frac{1}{(Pr)_{nf}}\theta''+f\theta'+N_b\phi'\theta'+N_t(\theta')^2=0 \qquad (15)$$

$$\phi''+(Sc)_{nf}\,f\,\phi'+\frac{N_t}{N_b}\theta''=0 \qquad (16)$$

with the corresponding boundary conditions

$$\begin{aligned} &f(0)=0, f'(0)=1,\ \theta(0)=1,\ \ \phi(0)=1 \\ &\qquad f'(\infty)=0,\ \theta(\infty)=0\ ,\ \phi(\infty)=0 \end{aligned} \qquad (17)$$

Here primes denote differentiation with respect to η and ν_{nf} and α_{nf} are the kinematic viscosity and thermal diffusivity of the nanofluid respectively. Also, $(Pr)_{nf}$, $(Sc)_{nf}$, N_b and N_t denote Prandtl number, Schmidt number, Brownian motion parameter and thermophoresis parameter respectively. Here, $(Re_x)_{nf}$ is the local Reynolds number, λ is the mixed convection parameter, λ^* is the modified mixed convection parameter, $(Gr_x)_{nf}$ is the local Grashof number and $(Gc_x)_{nf}$ is the modified local Grashof number of the nanofluid.

NUMERICAL ANALYSIS

The set of nonlinear differential equations subject to the boundary conditions constitute the nonlinear boundary value problem. The above boundary value problem is converted into an initial value problem by shooting method. As the ordinary methods fail in order to solve the system of transformed equations together with the asymptotic boundary conditions, a special iteration technique is used. Equations are solved numerically subject to using Fourth-Order Runge-Kutta based shooting method along with Nachtsheim-Swigert iteration scheme for satisfaction of asymptotic boundary conditions. Initial guesses for the values of $f''(0)$ and $\theta'(0)$ are made to initiate the shooting process and these initial guesses are made taking into account of convergency and numerical results are obtained for several values of the physical parameter $(Pr)_{nf}$.

RESULTS

The mixed convection problem associated with steady, non-linear, two-dimensional laminar flow of nanofluid over an inclined stretching sheet is thoroughly studied and numerical results are obtained.

For $\alpha = 0°$, the results reduce to that of Khan and Pop (2010) with regard to temperature and concentration. Numerical computations are carried out for various values of physical parameters which are chosen as Nt = 0.1, 0.2, 0.3, 0.4, 0.5 , α = 0°, 30°, 45° and 60° and Pr = 6.06 (copper-water nanofluid) for fixed values of Sc = 10, Nb = 0.1, λ = 0.5 and λ* = 0.5 and the results are shown graphically.It needs to be noted that the ordinary notations of Pr and Sc are used for easiness of representation instead of $(Pr)_{nf}$ and $(Sc)_{nf}$.

Fig.1 and Fig.2 demonstrate the comparison graphs with the earlier results. Fig.1 shows that when the angle of inclination α = 0°, the temperature distribution is in perfect agreement with that of Khan and Pop (2010).Also from Fig.2, it is clear that the concentration distribution is exactly identical to those by Khan and Pop (2010).

The effect of inclination angle on dimensionless velocity for specific parameters for copper-water nanofluid is portrayed through Fig.3. It is observed that when the angle of inclination α increases, dimensionless velocity increases. It is also elucidated that the thickness of the momentum boundary layer increases for the increase in the angle of inclination.

Fig.4 displays the effect of inclination angle α on temperature distribution for specified parameters for copper-water nanofluid. When α increases, temperature decreases. However, the change is not significant. Effect of inclination angle α on concentration distribution for specific parameters for copper-water nanofluid is portrayed in Fig.5. It is seen that as α increases, concentration decreases. Its behaviour is similar to the effect of temperature.

Fig.6 depicts the effect of Prandtl number on dimensionless velocity for the case of copper-water nanofluid. When Pr increases, non-dimensional velocity decreases. For copper-water nanofluid, it is shown through Fig.7 that dimensionless temperature decreases with increasing Pr. Also, Fig.8 portrays that when Pr increases, dimensionless concentration increases to start with and later, as we move away from the wall it has a different trend.

Numerical values of non-dimensional skin friction coefficient for different values of α and Nt is clearly shown in Table.4 which display the non-dimensional skin friction coefficient for different angles of inclination for copper-water nanofluid. It is observed that as α increases, non-dimensional skin friction increases. Further it is seen that for increasing values of Nt, non-dimensional skin friction coefficient increases for copper-water nanofluid when α is greater than zero.

In Table.5, dimensionless heat transfer rates for copper-water nanofluid for different inclination angles are depicted. As α increases, dimensionless heat transfer rate increases. The dimensionless heat transfer rates decreases with increasing Nt.

It is seen from Table.6 that as α increases, dimensionless concentration rates increase for fixed values of Pr, Sc, Nb, λ and λ* for copper-water nanofluid. The dimensionless concentration rates increase with the increase in the value of Nt.

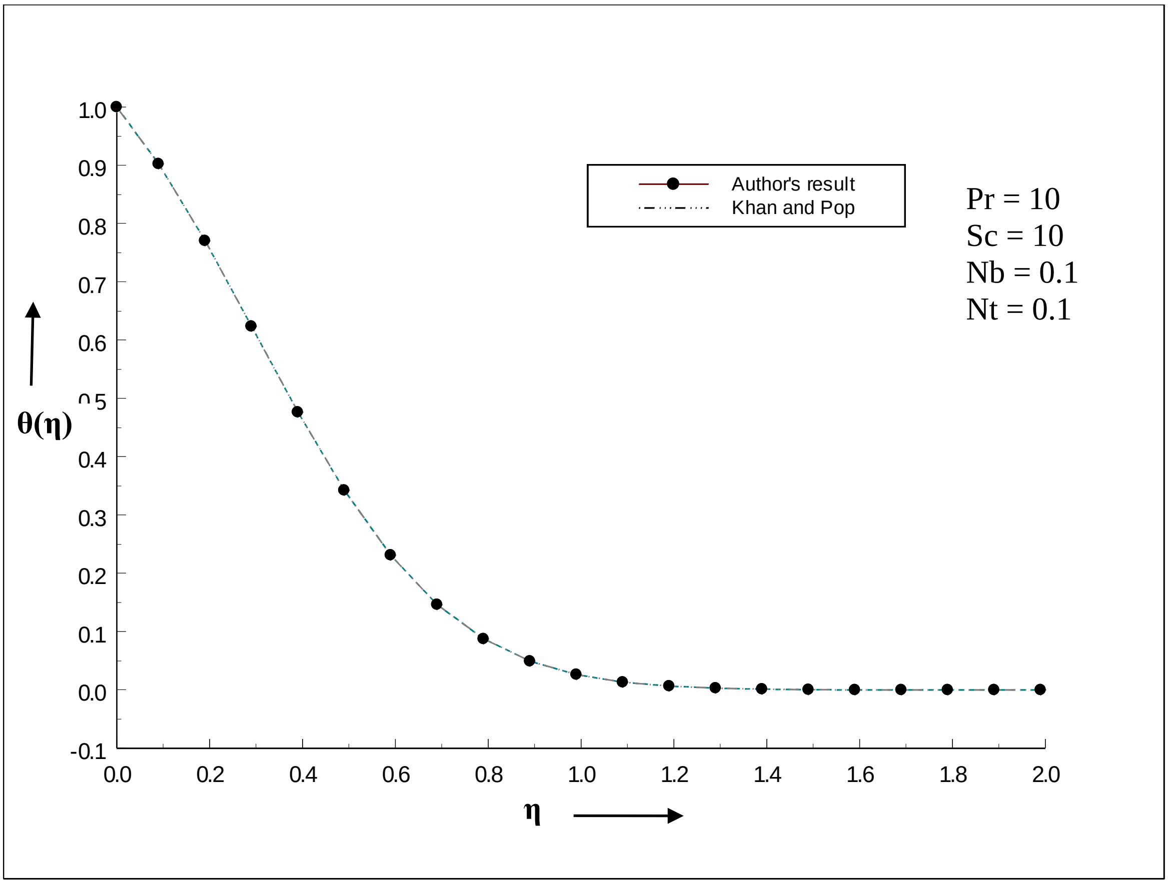

Fig.1 Comparison graph for temperature distribution when the angle of inclination α = 0°

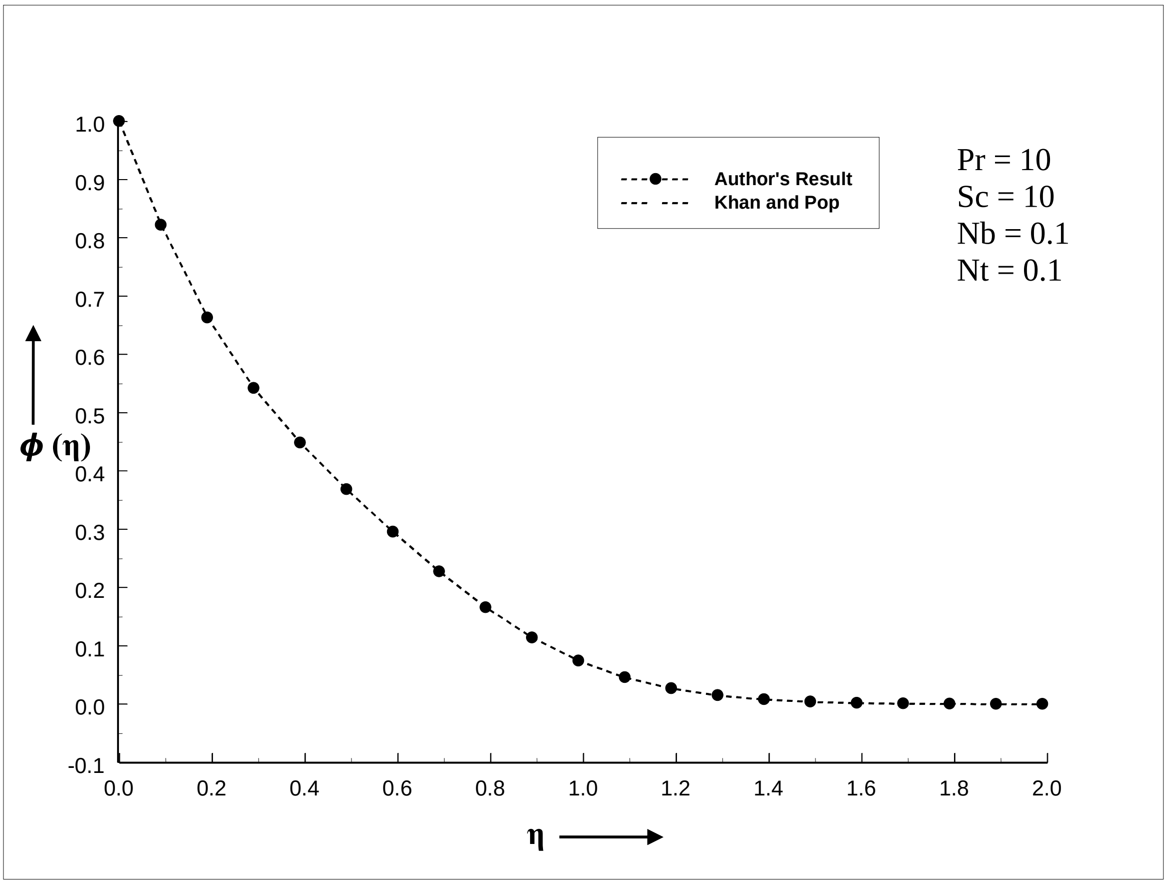

Fig.2 Comparison graph for concentration distribution when the angle of inclination α = 0°

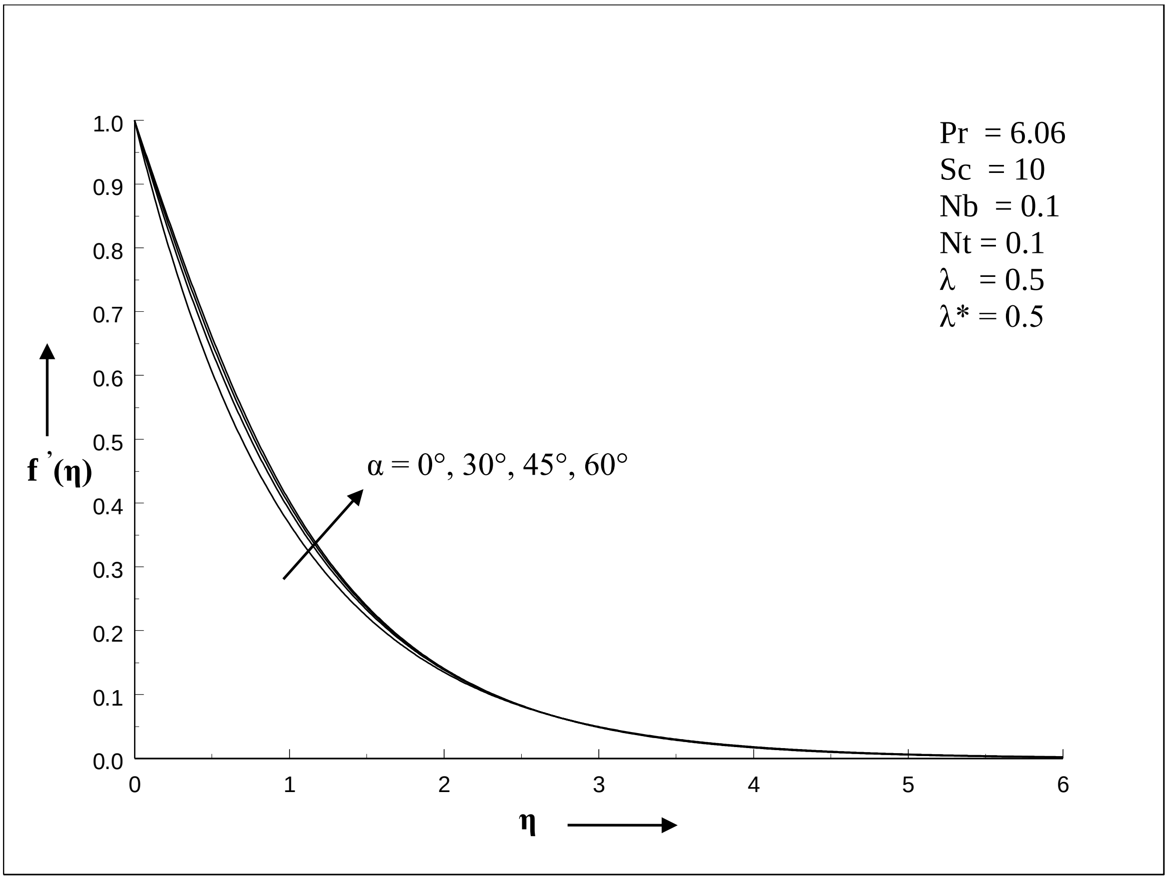

Fig.3 Effect of inclination angle over dimensionless velocity for specified parameters for copper-water nanofluid

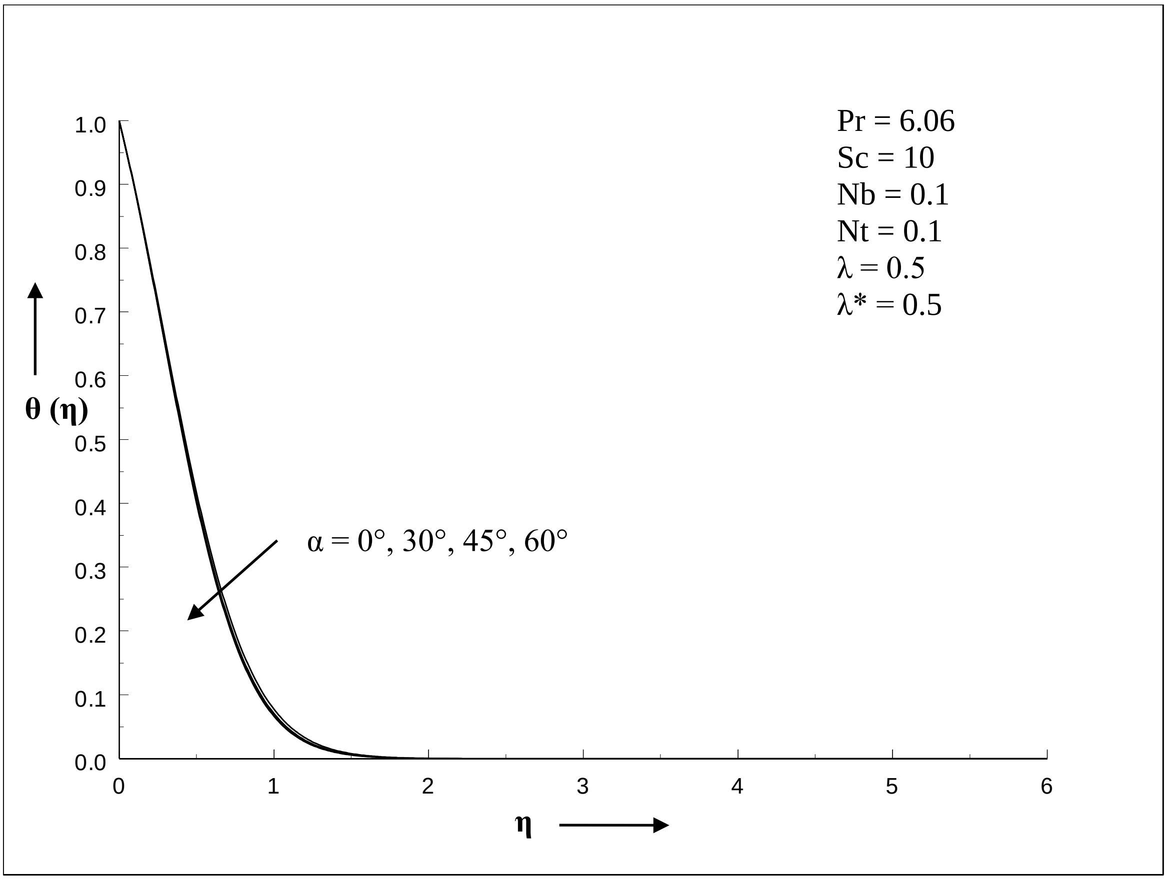

Fig.4 Effect of inclination angle over temperature distribution for specified parameters for copper-water nanofluid

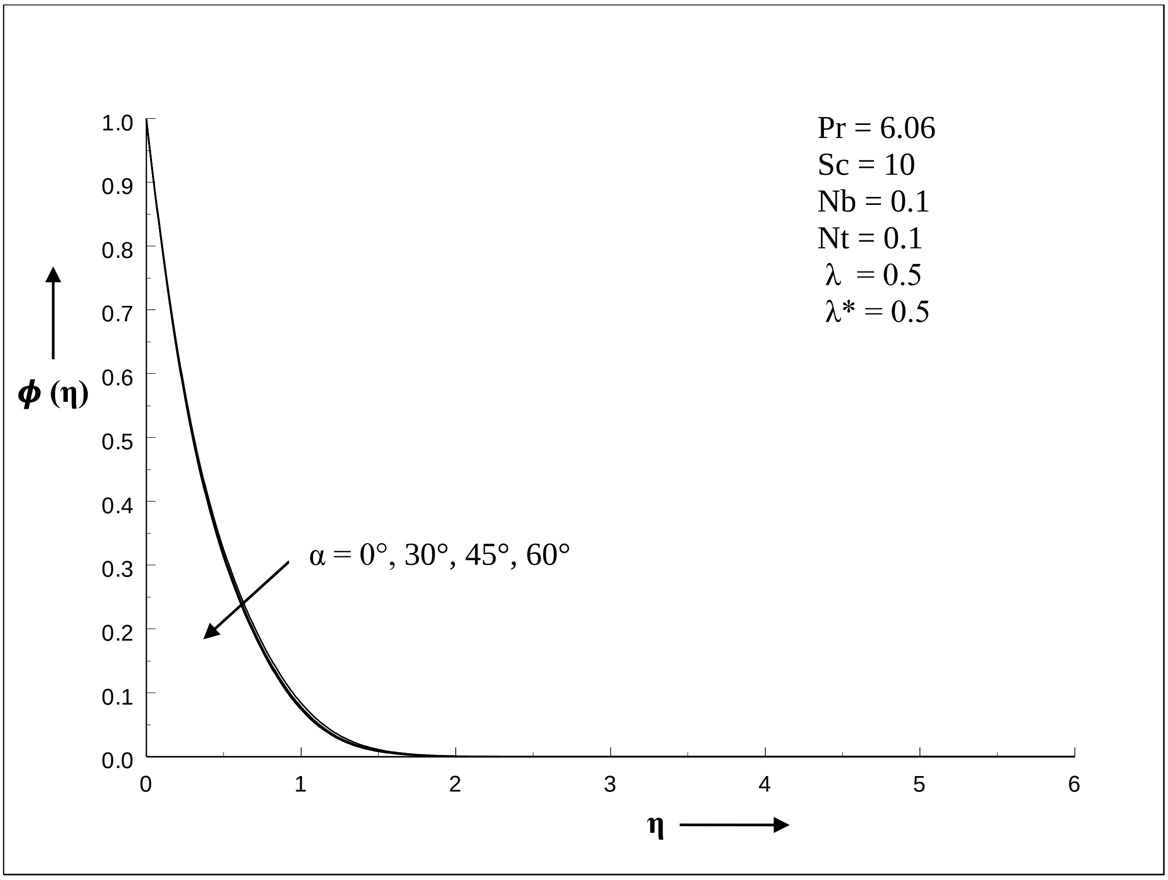

Fig.5 Effect of inclination angle over concentration distribution for specified parameters for copper-water nanofluid

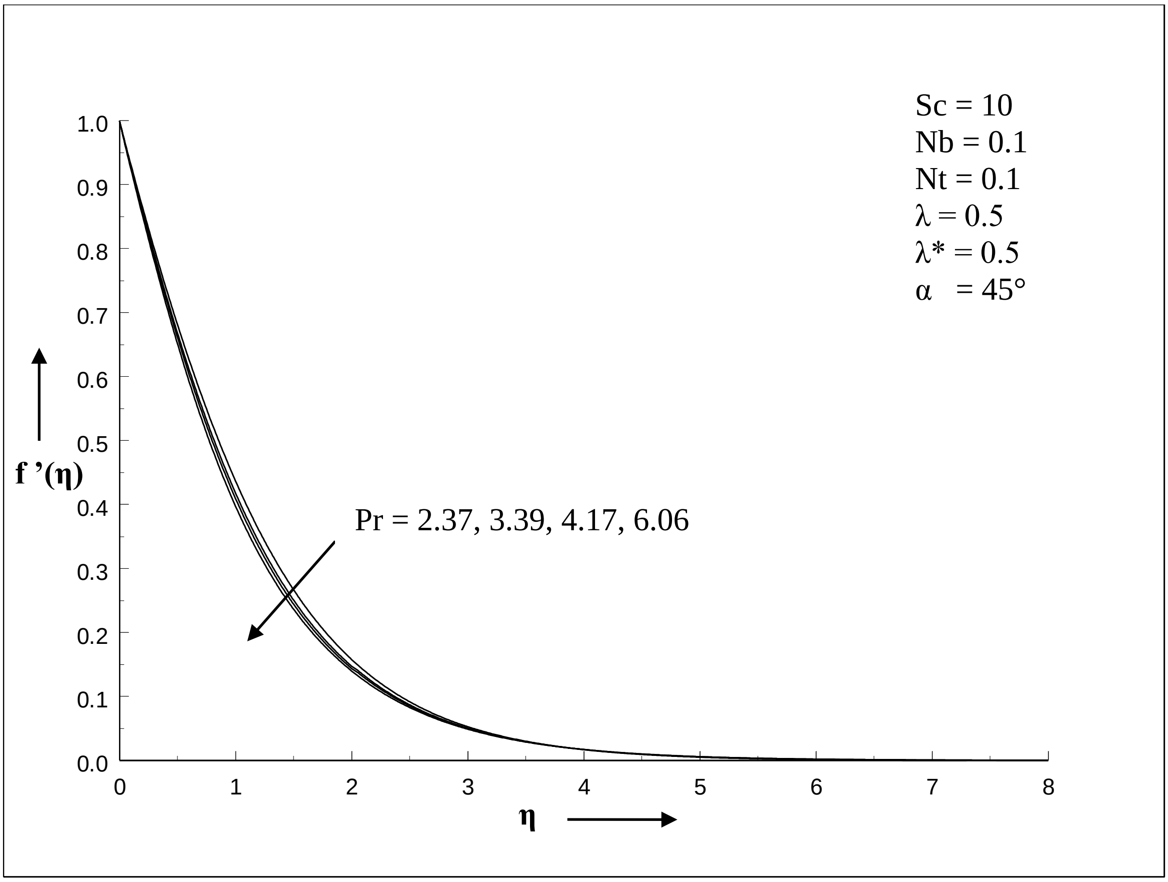

Fig.6 Effect of Pr over dimensionless velocity for specified parameters for copper-water nanofluid

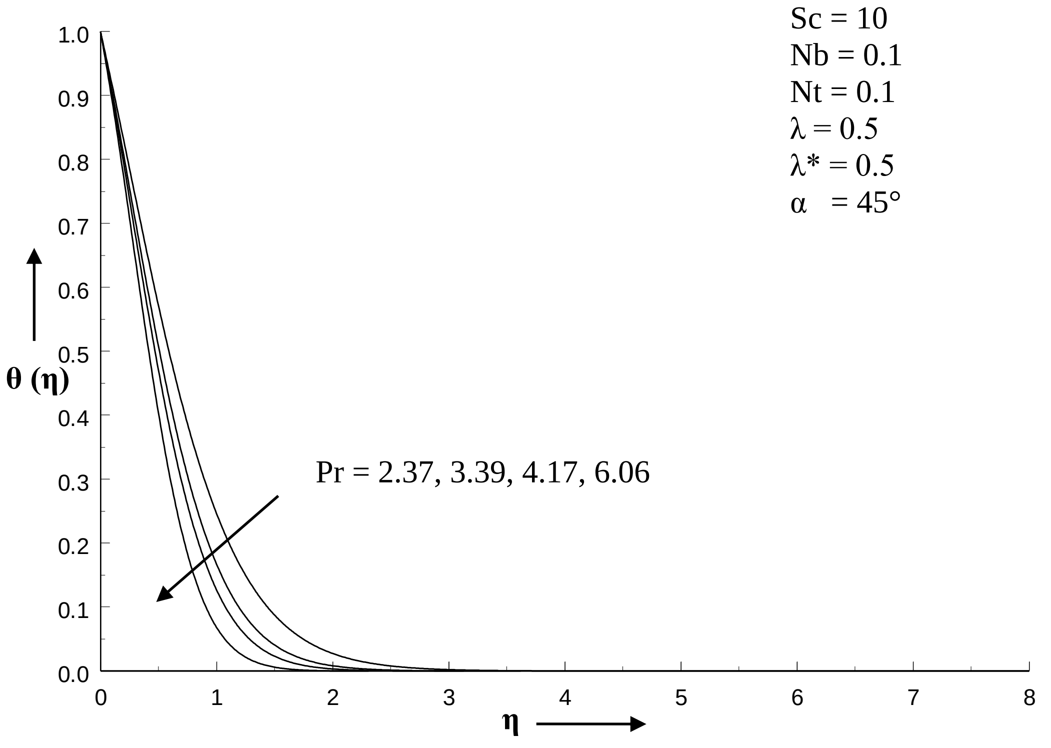

Fig.7 Effect of Pr over temperature distribution for specified parameters for copper-water nanofluid

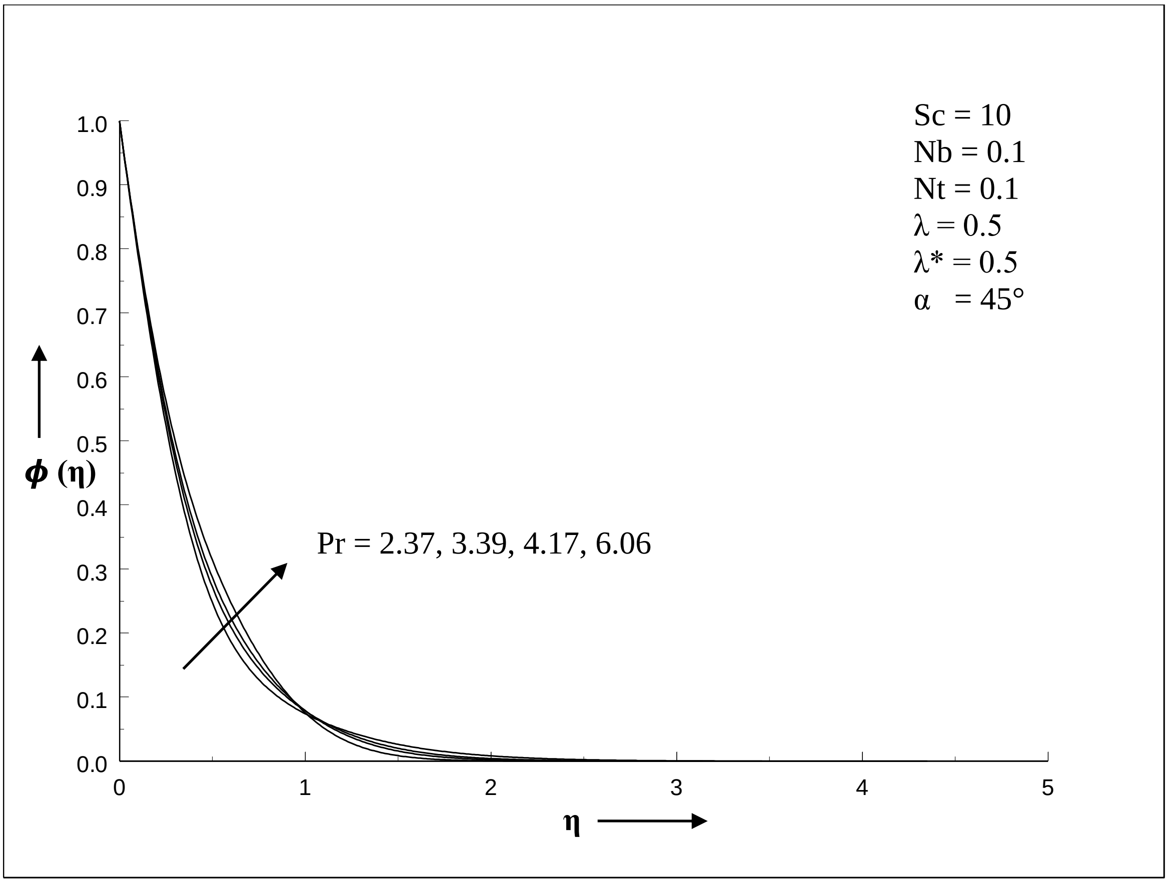

Fig.8 Effect of Pr over concentration distribution for specified parameters for copper-water nanofluid

Table. 2

Variation in $\frac{-1}{(1-\phi)^{2.5}} f''(0)$ for copper-water nanofluid for different α and Nt when Pr = 6.06, ϕ = 0.01, Sc = 10, Nb = 0.1, λ = 0.5, λ* = 0.5

α	**Nt**	$f''(0)$	$\frac{-1}{(1-\phi)^{2.5}} f''(0)$
0°	0.1	1.0000	1.0254
	0.2	1.0000	1.0254
	0.3	1.0000	1.0254
	0.4	1.0000	1.0254
	0.5	1.0000	1.0254
30°	0.1	0.8399	0.8551
	0.2	0.8202	0.8411
	0.3	0.8135	0.8342
	0.4	0.8015	0.8219
	0.5	0.7900	0.8101
45°	0.1	0.7737	0.7934
	0.2	0.7559	0.7751
	0.3	0.7384	0.7572
	0.4	0.7221	0.7405
	0.5	0.7063	0.7243
60°	0.1	0.7242	0.7426
	0.2	0.7027	0.7206
	0.3	0.6820	0.6994
	0.4	0.6621	0.6789
	0.5	0.6431	0.6595

Table. 3

Variation in $-(k_{nf} / k_f)\ \theta'(0)$ for copper-water nanofluid for different α and Nt when Pr = 6.06, $\phi = 0.01$, Sc = 10, Nb = 0.1, $\lambda = 0.5$, $\lambda^* = 0.5$

α	Nt	$-\theta'(0)$	$-(k_{nf} / k_f)\ \theta'(0)$
0°	0.1	0.9599	1.0538
	0.2	0.7800	0.8562
	0.3	0.6300	0.6916
	0.4	0.5300	0.5818
	0.5	0.4400	0.4830
30°	0.1	0.9699	1.0648
	0.2	0.7900	0.8672
	0.3	0.6400	0.7025
	0.4	0.5400	0.5928
	0.5	0.4500	0.4940
45°	0.1	0.9799	1.0758
	0.2	0.7910	0.8683
	0.3	0.6500	0.7135
	0.4	0.5400	0.5928
	0.5	0.4530	0.4973
60°	0.1	0.9829	1.0791
	0.2	0.7930	0.8705
	0.3	0.6510	0.7146
	0.4	0.5405	0.5934
	0.5	0.4550	0.4994

Table. 4

Variation in $-\frac{(D_B)_{nf}}{(D_B)_f}\phi'(0)$ for copper-water nanofluid for different α and Nt when Pr = 6.06, ϕ = 0.01, Sc = 10, Nb = 0.1, λ = 0.5, λ* = 0.5

α	**Nt**	$\phi'(0)$	$-\frac{(D_B)_{nf}}{(D_B)_f}\phi'(0)$
0°	0.1	2.0499	1.9961
	0.2	2.0550	2.0010
	0.3	2.1400	2.0837
	0.4	2.3008	2.2404
	0.5	2.4790	2.4138
30°	0.1	2.0880	2.0331
	0.2	2.0980	2.0428
	0.3	2.2088	2.1508
	0.4	2.3938	2.3309
	0.5	2.5990	2.5306
45°	0.1	2.1000	2.0448
	0.2	2.1019	2.0466
	0.3	2.2349	2.1762
	0.4	2.4275	2.3637
	0.5	2.6429	2.5734
60°	0.1	2.1099	2.0545
	0.2	2.1154	2.0598
	0.3	2.2524	2.1932
	0.4	2.4521	2.3877
	0.5	2.6741	2.6038

5.CONCLUSION

The problem of laminar mixed convection boundary layer flow of an incompressible viscous nanofluid resulting from an inclined stretching surface has been investigated numerically. Numerical results for velocity and temperature distribution are obtained for various values of physical parameters. The variations in non-dimensional skin friction coefficient, Nusselt and Sherwood numbers with Nb and Nt for various values of Pr and Sc are also presented by means of tables. It is found that the reduced Nusselt number and the reduced Sherwood number increases for increasing values of α and for fixed values of the parameters Pr, Sc, Nb, λ and λ^* considered for copper-water nanofluids.

When $\alpha = 0°$, our results are in excellent agreement with that of Khan and Pop (2010) for Pr = 10 and Sc = 10.

From all the numerical computations, the following main conclusions can be drawn from the present study.

- For copper-water nanofluid, as the inclination angle α increases, dimensionless velocity increases. Further dimensionless temperature and dimensionless concentration decrease with increasing α.

- For copper-water nanofluid, as the inclination angle α increases, non-dimensional skin friction coefficient, reduced Nusselt number Nur and reduced Sherwood number Shr increases respectively.

- For increasing values of thermphoresis number Nt for the copper-water nanofluid, for fixed inclination angle α, non-dimensional skin friction coefficient increases, reduced Nusselt number Nur decreases and reduced Sherwood number Shr increases.

- For increasing Pr, dimensionless velocity and dimensionless temperature decreases in the case of copper-water nanofluid.

- In the case of copper-water nanofluid flow, dimensionless concentration increases up to certain distance from the plate and thereafter it gets decreased.

6. CONTRIBUTION TO HUMANITY AND FUTURE SCOPE

One of the goals of theoretical research on nanofluids is to develop a theory of nanofluids to explain how nanoparticles change the thermal properties of nanofluids. A theory of nanofluids would also provide a theoretical foundation for physics and chemistry based predictive models.

Numerous studies conducted on nanofluids have made scientific breakthrough not only in discovering unexpected thermal properties of nanofluids, but also in proposing new mechanisms behind the enhanced thermal properties of nanofluids and thus identifying unusual opportunities to develop them as next generation coolants for computers and safe coolants for nuclear reactors. Applied research in nanofluids has demonstrated in the laboratory that nanoparticles can be used to enhance the thermal conductivity and heat transfer performance of conventional heat transfer fluids. Some researchers took the concept one step into practical applications and demonstrated the ability of nanofluids to improve the performance of real world devices and systems such as automatic transmissions. Thus, nanofluid research has made the initial transition from our laboratory to industrial research laboratories. This important work has provided guidance as to the right direction, the first step in the development of commercial nanofluid technology. With continued collaboration between basic and applied nanofluid researchers in academia and industry on thermal properties, performance, theory, mechanisms, modeling, development and eventual commercialization of nanofluids, nanofluid research is expected to bring breakthroughs in nanotechnology-based cooling technology and have a strong impact on a wide range of engineering and biomedical applications. In future, promising nanofluids should be studied not only under real- world conditions of use, but also over a longer period of time. Nanofluids will be in number one position for the contribution of the humanity to the newer horizons. Concluding, we can remark that surely, coming days are for the nanofluids.

PUBLICATIONS

1. "Boundary layer flow of nanofluid past an inclined stretching sheet", International Journal of Theoretical and Computational Mathematics,(ISSN-2395-6607), Vol 2, No.2, pp7-13.
2. "Boundary layer flow of nanofluid past an inclined stretching sheet" Proceedings of Fifth International Conference on" Advances in Mechanical and Robotics Engineering" AMRE'17 (ISBN :978-1-63248-123-8 doi: 10.15224/978-1-63248-123-8-38)

PRESENTATIONS

1. Presented the paper entitled "Nanofluids: Coolants with novel properties and diverse applications" in International conference in Mathematics and Computer Science on 13-14 December , 2016 at St.Thomas' College(Autonomous), Thrisssur, Kerala, India.

2. Presented the paper entitled "Boundary layer flow of nanofluid past an inclined stretching sheet" , in Fifth International Conference on" Advances in Mechanical and Robotics Engineering" AMRE'17, organized by Institute of Research Engineers and Doctors on 27-28 May, 2017, at Roma Eur, Rome, Itay.

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